

# 2.8 Test Report

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1	. CONTENTS	
2	INTRODUCTION	5
2.1.	Summary of developmental validation tests	5
2.2.	Summary of extended output tests	6
3	ACCEPTANCE CRITERIA	10
3.1.	Microsoft Excel precision	10
3.2.	Normal density comparison	11
3.3.	Cumulative Normal approximation	18
3.4.	Unit test exponential function	23
3.5.	Unit test Gamma distribution	24
3.6.	Variables with stochastic elements	25
4.	RESULTS OF DEVELOPMENTAL VALIDATION	26
4.1.	Pre checks for missing stutter peaks	26
4.2.	Modelling drop-in peaks in stutter positions	27
4.3.	Smart start	27
4.4.	Expected peak heights given mass parameters	28
4.4.1.	Expected peak heights using stutter regression file	29
4.4.2.	Expected peak heights using stutter exceptions file	32
4.5.	Log(likelihood) values	32
4.5.1.	Taylor quantum effect, no dropout	32
4.5.2.	Taylor quantum effect, peaks below z (dropped peaks)	34
4.6.	Probabilities and expected stutter peak heights given saturated alleles	36
4.7.	Shifted log normal for composite peaks	39
4.8.	Expected peak heights for replicate profiles	41
4.8.1.	Replicates conditioning on an individual whose alleles has 'dropped out' of the profil	e45
4.9.	Summation of probabilities	47
4.10.	Locus specific amplification efficiency penalties	53
4.11.	Penalties	55
4.11.1.	Allele and stutter variance penalties	55
4.11.2.	Drop-in penalty	57
4.11.3.	Mx prior penalties	58
4.12.	Replicating Genotype Weights	59
4.13.	Likelihood ratios	61
4.13.1.	Posterior mean allele frequencies	61
4.13.2.	<i>LR</i> point estimates (sub-sub-source <i>LR</i> )	61
4.13.3.	Sub-source LR	65



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4.13.4. <i>LR</i> point estimates with varying theta values	67
4.13.5. Relatives <i>LR</i> s	69
4.13.6. Unified <i>LR</i>	79
4.13.7. Stratified <i>LR</i>	80
4.14. Highest posterior density	81
4.14.1. HPD for unrelated and related <i>LR</i> s	81
4.14.2. Stratified HPD for unrelated and related <i>LR</i> s	82
4.14.2.1. HPD unit test – allele frequency resampling	82
4.14.2.2. HPD unit test – weight resampling	83
4.14.3. Sampling theta values during HPD	83
4.15. Gaussian walk	84
4.16. Post burn-in diagnostics	92
4.16.1. Gelman-Rubin convergence diagnostic	92
4.16.2. Effective sample size	93
4.16.3. Effective sample size thinning	93
4.16.4. Total iterations	94
4.16.5. Acceptance rate	95
4.16.6. Average log(likelihood)	95
4.16.7. Template	95
4.16.8. Mixture proportions	98
4.16.9. Degradation	99
4.16.10. Allele, stutter, and LSAE variance constants	100
4.16.11. Locus Specific Amplification Efficiencies	101
4.16.12. Replicate efficiency	101
4.16.13. Kit efficiency	102
4.17. Mix to Mix function	102
4.18. Top-down approach	107
4.19. <i>H<sub>d</sub></i> true tester	110
4.20. Model Maker	112
4.21. Variable number of contributors function	113
4.21.1. Hyper-rectangle	114
4.21.2. MC average	118
4.21.3. F(Omega)	121
4.21.4. Marginal likelihoods	121
4.22. varNOC likelihood ratios	122
4.22.1. varNOC sub-source <i>LR</i> S	122
4.22.2. varNOC HPD <i>LR</i>	126



	STRmix™ V2.8 Test Report
	Issued by Institute of Environmental Science and Research Limited
	Date of Issue: 29 September 2020
4.23. CODIS report	126
4.23.1. CODIS match threshold	128
4.24. Precision	129
4.25. Sensitivity and Specificity	136
5. REFERENCES	

# 2. INTRODUCTION

This document discusses the testing and developmental validation carried out for STRmix<sup>™</sup> version 2.8 prior to commercial release. Developmental validation is carried out using acceptance/qualification testing to determine whether or not each element of the system satisfies pre-determined acceptance criteria.

#### 2.1. Summary of developmental validation tests

The following functionality and outputs from STRmix<sup>™</sup> were verified by hand as part of the developmental validation tasks:

- 1. Pre checks for missing stutter
- 2. Smart start
- 3. Expected allele and stutter heights given mass parameters using stutter regression and exceptions files
- 4. Expected peak heights of dropout or 'Q' alleles given mass parameters
- 5. Probabilities given expected and observed peak heights including those below the analytical threshold
- 6. Locus specific amplification efficiency penalties
- 7. Summation of probabilities for each allele in a locus and across a profile
- 8. Summation of probabilities across multiple replicate profiles and multiple kits
- 9. Informed priors on mixture proportion
- 10. LR values with and without assumed contributors
- 11. LR values with varying theta values
- 12. *LR* stratified and unified point estimates
- 13. *LR* HPD interval values including theta beta and allele frequency resampling
- 14. Gaussian walk
- 15. Gelman-Rubin statistic, ESS, weight resampling
- 16. Relatives calculations
- 17. *LR* Batcher, Batch Mode and *LR* from previous functionality
- 18. Drop-in function
- 19. Database search functionalities including familial matching
- 20. Variable number of contributors function and *LR*s
- 21. *Hd* true tester function
- 22. Mix to mix LRs
- 23. Model Maker.

Many calculations including the calculation of expected heights, penalties, and *LR* values were conducted in Excel. A summary of the developmental validation results will be discussed in turn for each comparison undertaken. Note that this is only a small subset of all the testing activities that were undertaken during the STRmix<sup>™</sup> development project.

# **STRmix**

#### 2.2. Summary of extended output tests

Extended output testing is the "by hand" testing of the biological and statistical models within STRmix. The extended outputs contain all the outputs needed to replicate the models. For example for an interpretation these contain the genotype combinations, mass parameters, observed and expected peak heights, peak height and LSAE variances, and all penalties including peak, variances, drop-in, Mx priors, and their sum per iteration.

A list of all the samples and functions tested is given on the next page. All results were as expected. The Excel sheets are saved within the extended outputs folder of each run folder which are saved in the following location:

V:\STRmix v2.8\v2.8 Science\Extendeds 2.8.0.23

The extended outputs were checked in V2.8.0.23 which was the version with the final science changes (defined as where a change affects the seed). Release version STRmix 2.8.0.34 was compared against builld 23 using the standard set of configs and no changes in these results were observed.

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Profile	burn- in	log p drop	log p not drop	SLN	smart start	drop- in	TAP + <i>E</i> a	SRa	c² an <i>k</i> ²	Mx priors	Peaks > s	LSAE var	LSAE penalty	Total penalty
Ex1A drop in_gamma	-	у	у	у	у	у	у	у	у	N/A	N/A	у	у	у
Ex1A drop in_uniform	-	у	у	у	у	у	-	-	у	N/A	N/A	у	у	у
Ex1A GF_extended	-	У	У	у	У	N/A	у	у	У	N/A	N/A	У	У	У
Ex1A saturated_GF 3500 sat -5000	-	у	у	у	у	N/A	у	у	у	N/A	у	у	у	у
Ex1C_assumed extended	-	У	у	у	у	N/A	у	у	у	N/A	N/A	у	у	-
extended_reps (Ex1E)	-	у	у	у	у	N/A	у	у	У	N/A	N/A	-	у	у
multi kit_reps	-	У	У	у	У	N/A	у	у	y	N/A	N/A	У	У	У
NIST case 1_multikit assumed	-	Y	у	у	у	N/A	у	у	у	N/A	N/A	-	у	у
NIST mix 3_conditioned two	-	у	у	у	у	N/A	у	у	у	N/A	N/A	Y	у	у
NIST mix 3_three person	-	у	у	у	у	N/A	-	-	у	N/A	N/A	-	у	у



Page 7 of 141

Profile	burn- in	log p drop	log p not drop	SLN	smart start	drop- in	TAP + <i>E</i> a	SR <sub>a</sub>	c² an <i>k</i> ²	Mx priors	Peaks > s	LSAE var	LSAE penalty	Total penalty
NIST 3 3p_mx priors	-	у	у	у	у	N/A	у	у	у	у	N/A	-	у	у
NIST case 1_multikit varNOC	-	у	у	у	у	N/A	Y	Y	у	N/A	N/A	у	у	у
TC 2p_reps assumed VF02	-	N/A	у	у	у	N/A	Y	Y	у	N/A	у	-	у	у
Multikit_multi dropin extended		у	у	у	N/A	у	-	-	у	N/A	N/A	-	у	-
Ex1A_stutter proprotion b-2000 AT D22 100 rfu	Y	У	у	у	у	N/A	-	-	у	N/A	N/A	у	у	у
Ex1A no stutter_extended	-	y (CNA log p)	у	у	у	N/A	-	-	у	N/A	N/A	-	у	у
Ex1C missing stutter_extended	y (only)	у	у	N/A	)	N/A	Y	Y	у	N/A	N/A	у	у	у
Ex1E as 2_conditioned	-	Y	Y	-	у	N/A	у	у	у	N/A	N/A	у	у	у



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Mx priors		Peaks > s	LSAE var	LSAE penalty	Total penalty
N/A	\ N/	N/A	Y	Y	у
N/A	N/	N/A	Y	Y	у
N/A	N/	N/A	у	у	у
N/A	N/	N/A	у	у	у
N/A	N/	N/A	-	-	у
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Page 9 of 141

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### 3. ACCEPTANCE CRITERIA

#### 3.1. Microsoft Excel precision

Microsoft Excel implements the IEEE 754 floating-point standard. Excel stores 15 significant digits of precision. For example, the number 1234567890123456 cannot be exactly represented if 15 digits of precision are used.

The IEEE 754 floating-point standard requires that numbers be stored in binary format. This means a conversion must occur before the numbers can be used in calculations. If the number can be represented exactly in floating-point format, then the conversion is exact. If not, then the conversion will result in a rounded value which will represent the original value. Numbers that appear exact in the decimal format may need to be approximated when converted to binary floating-point. For example, the fraction 1/10 can be represented in the decimal format as the rational number 0.1. However, 0.1 cannot be represented precisely in binary floating-point of finite precision. 0.1 becomes the repeating binary decimal 0.0001100110011', where the sequence 1100 repeats infinitely. This number cannot be represented in a finite amount of space and therefore in Excel, it is rounded down by approximately 2.8E-17 when it is stored.

The discrepancy between the STRmix<sup>™</sup> output and the Excel repeat is defined by, at least, the Excel precision.

#### **3.2. Normal density comparison**

The unit test for the normal distribution density method within STRmix<sup>TM</sup> [com.strmix.calculation.STRMath.calculateNormalDistributionDensity] over the range [-20,20] in increments of 0.01 with mean=0, var=1 was obtained. The output was compared with Excel using the norm.s.dist(X,0) function. The results appear Figure 3.1 Comparison of normal density STRmix<sup>TM</sup> and MS Exceland Table 3.1.

		Log(STRmix)-			Log(STRmix)-			Log(STRmix)-			Log(STRmix)-
Х	log(STRmix)	log(Excel)	Х	log(STRmix)	log(Excel)	Х	log(STRmix)	log(Excel)	х	log(STRmix)	log(Excel)
-20	-87.26	0	-10	-22.11	-1.4E-14	3.27E-13	-0.40	-8.3E-16	10	-22.11	-2.6E-13
-19.9	-86.39	-7.2E-13	-9.9	-21.68	-3.2E-14	0.1	-0.40	-1E-15	10.1	-22.55	-2.5E-13
-19.8	-85.53	-5.7E-13	-9.8	-21.25	-3.6E-14	0.2	-0.41	-3.9E-16	10.2	-22.99	-2.4E-13
-19.7	-84.67	-4.7E-13	-9.7	-20.83	0	0.3	-0.42	-8.3E-16	10.3	-23.44	-2.3E-13
-19.6	-83.82	-3.3E-13	-9.6	-20.41	-1.8E-14	0.4	-0.43	-2.2E-16	10.4	-23.89	-2.3E-13
-19.5	-82.97	-1.7E-13	-9.5	-20.00	-2.5E-14	0.5	-0.45	-1.2E-15	10.5	-24.34	-2.2E-13
-19.4	-82.12	-7.1E-14	-9.4	-19.59	-2.8E-14	0.6	-0.48	-3.3E-16	10.6	-24.80	-2.1E-13
-19.3	-81.28	-7.5E-13	-9.3	-19.18	0	0.7	-0.51	-1.1E-15	10.7	-25.26	-2E-13
-19.2	-80.45	-6.4E-13	-9.2	-18.78	-1.1E-14	0.8	-0.54	-1.4E-15	10.8	-25.73	-2E-13
-19.1	-79.62	-5E-13	-9.1	-18.38	-1.4E-14	0.9	-0.57	-8.9E-16	10.9	-26.20	-1.8E-13
-19	-78.79	-3.6E-13	-9	-17.99	-2.5E-14	1	-0.62	-3.6E-15	11	-26.67	-1.8E-13

#### Table 3.1 A comparison of log(STRmix) vs log(STRmix)-log(Excel)



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-18.9	-77.97	-2.3E-13	-8.9	-17.60	-2.8E-14	1.1	-0.66	-3.7E-15	11.1	-27.15	-1.7E-13
-18.8	-77.15	-8.5E-14	-8.8	-17.21	0	1.2	-0.71	-5.3E-15	11.2	-27.64	-1.5E-13
-18.7	-76.33	-7.8E-13	-8.7	-16.83	-7.1E-15	1.3	-0.77	-5.2E-15	11.3	-28.13	-1.5E-13
-18.6	-75.52	-6.7E-13	-8.6	-16.46	-1.8E-14	1.4	-0.82	-5.2E-15	11.4	-28.62	-1.4E-13
-18.5	-74.72	-5.1E-13	-8.5	-16.09	-2.5E-14	1.5	-0.89	-5.2E-15	11.5	-29.12	-1.4E-13
-18.4	-73.92	-4E-13	-8.4	-15.72	-3.4E-14	1.6	-0.95	-6.9E-15	11.6	-29.62	-1.2E-13
-18.3	-73.12	-2.8E-13	-8.3	-15.36	0	1.7	-1.03	-6.4E-15	11.7	-30.12	-1.1E-13
-18.2	-72.33	-1.6E-13	-8.2	-15.00	-1.2E-14	1.8	-1.10	-6.4E-15	11.8	-30.63	-9.9E-14
-18.1	-71.54	-4.3E-14	-8.1	-14.65	-1.8E-14	1.9	-1.18	-7.1E-15	11.9	-31.15	-9.2E-14
-18	-70.75	-7E-13	-8	-14.30	-2.5E-14	2	-1.27	-7.1E-15	12	-31.67	-8.9E-14
-17.9	-69.98	-5.5E-13	-7.9	-13.95	-3.4E-14	2.1	-1.36	-6.4E-15	12.1	-32.19	-7.8E-14
-17.8	-69.20	-4.4E-13	-7.8	-13.61	-5.3E-15	2.2	-1.45	-4.4E-15	12.2	-32.72	-5.7E-14
-17.7	-68.43	-3.1E-13	-7.7	-13.27	-1.2E-14	2.3	-1.55	-2E-15	12.3	-33.25	-5E-14
-17.6	-67.66	-2E-13	-7.6	-12.94	-2.1E-14	2.4	-1.65	-1E-14	12.4	-33.79	-5E-14
-17.5	-66.90	-8.5E-14	-7.5	-12.61	-2.8E-14	2.5	-1.76	-8.9E-15	12.5	-34.33	-2.8E-14
-17.4	-66.14	-7E-13	-7.4	-12.29	0	2.6	-1.87	-8.4E-15	12.6	-34.87	-2.1E-14

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-17.3	-65.39	-6E-13	-7.3	-11.97	-7.1E-15	2.7	-1.98	-7.1E-15	12.7	-35.42	0
-17.2	-64.64	-4.8E-13	-7.2	-11.66	-1.4E-14	2.8	-2.10	-2.2E-15	12.8	-35.98	-5.5E-13
-17.1	-63.90	-3.6E-13	-7.1	-11.35	-2E-14	2.9	-2.23	-1.2E-14	12.9	-36.53	-5.5E-13
-17	-63.15	-2.4E-13	-7	-11.04	-2.5E-14	3	-2.35	-9.8E-15	13	-37.10	-5.3E-13
-16.9	-62.42	-1.1E-13	-6.9	-10.74	-1.8E-15	3.1	-2.49	-7.1E-15	13.1	-37.66	-5.3E-13
-16.8	-61.69	-7.1E-15	-6.8	-10.44	-7.1E-15	3.2	-2.62	-4.4E-15	13.2	-38.23	-5.2E-13
-16.7	-60.96	-6.2E-13	-6.7	-10.15	-1.2E-14	3.3	-2.76	-1.3E-15	13.3	-38.81	-5E-13
-16.6	-60.24	-4.9E-13	-6.6	-9.86	-2.3E-14	3.4	-2.91	-1.6E-14	13.4	-39.39	-5E-13
-16.5	-59.52	-3.8E-13	-6.5	-9.57	-2.5E-14	3.5	-3.06	-9.8E-15	13.5	-39.97	-4.9E-13
-16.4	-58.80	-2.6E-13	-6.4	-9.29	-1.8E-15	3.6	-3.21	-6.7E-15	13.6	-40.56	-4.8E-13
-16.3	-58.09	-1.5E-13	-6.3	-9.02	-1.1E-14	3.7	-3.37	-3.1E-15	13.7	-41.16	-4.8E-13
-16.2	-57.39	-5E-14	-6.2	-8.75	-1.8E-14	3.8	-3.53	-8.9E-16	13.8	-41.75	-4.6E-13
-16.1	-56.69	-6.2E-13	-6.1	-8.48	-2.1E-14	3.9	-3.70	-1.5E-14	13.9	-42.35	-4.5E-13
-16	-55.99	-5.3E-13	-6	-8.22	0	4	-3.87	-1.2E-14	14	-42.96	-4.5E-13
-15.9	-55.30	-5.4E-13	-5.9	-7.96	-6.2E-15	4.1	-4.05	-7.1E-15	14.1	-43.57	-4.3E-13
-15.8	-54.61	-5.5E-13	-5.8	-7.70	-1.1E-14	4.2	-4.23	-3.6E-15	14.2	-44.18	-4.3E-13

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-15.7	-53.92	-5.7E-13	-5.7	-7.45	-1.6E-14	4.3	-4.41	-8.9E-16	14.3	-44.80	-4.1E-13
-15.6	-53.24	-5.8E-13	-5.6	-7.21	-2E-14	4.4	-4.60	-1.3E-14	14.4	-45.43	-4.1E-13
-15.5	-52.57	-5.8E-13	-5.5	-6.97	-2.7E-15	4.5	-4.80	-1.2E-14	14.5	-46.05	-3.8E-13
-15.4	-51.90	-5.9E-13	-5.4	-6.73	-6.2E-15	4.6	-4.99	-7.1E-15	14.6	-46.69	-3.8E-13
-15.3	-51.23	-6E-13	-5.3	-6.50	-1.1E-14	4.7	-5.20	-8.9E-16	14.7	-47.32	-3.8E-13
-15.2	-50.57	-6.2E-13	-5.2	-6.27	-1.7E-14	4.8	-5.40	-2E-14	14.8	-47.96	-3.6E-13
-15.1	-49.91	-6.3E-13	-5.1	-6.05	-2.1E-14	4.9	-5.61	-1.7E-14	14.9	-48.61	-3.5E-13
-15	-49.26	-6.4E-13	-5	-5.83	-5.3E-15	5	-5.83	-9.8E-15	15	-49.26	-3.3E-13
-14.9	-48.61	0	-4.9	-5.61	-8E-15	5.1	-6.05	-5.3E-15	15.1	-49.91	-3.3E-13
-14.8	-47.96	-1.4E-14	-4.8	-5.40	-1.2E-14	5.2	-6.27	-8.9E-16	15.2	-50.57	-3.1E-13
-14.7	-47.32	-2.8E-14	-4.7	-5.20	-1.5E-14	5.3	-6.50	-1.9E-14	15.3	-51.23	-2.8E-13
-14.6	-46.69	-4.3E-14	-4.6	-4.99	-2.4E-14	5.4	-6.73	-1.5E-14	15.4	-51.90	-2.8E-13
-14.5	-46.05	-5.7E-14	-4.5	-4.80	-6.2E-15	5.5	-6.97	-1.2E-14	15.5	-52.57	-2.8E-13
-14.4	-45.43	-7.1E-14	-4.4	-4.60	-8.9E-15	5.6	-7.21	-4.4E-15	15.6	-53.24	-2.7E-13
-14.3	-44.80	-7.8E-14	-4.3	-4.41	-1.2E-14	5.7	-7.45	0	15.7	-53.92	-2.6E-13
-14.2	-44.18	-8.5E-14	-4.2	-4.23	-1.5E-14	5.8	-7.70	-2.1E-14	15.8	-54.61	-2.4E-13

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-14.1	-43.57	-1.1E-13	-4.1	-4.05	-8.9E-16	5.9	-7.96	-1.5E-14	15.9	-55.30	-2.2E-13
-14	-42.96	-1.3E-13	-4	-3.87	-4.4E-15	6	-8.22	-8.9E-15	16	-55.99	-2.3E-13
-13.9	-42.35	-1.4E-13	-3.9	-3.70	-9.8E-15	6.1	-8.48	-1.8E-15	16.1	-56.69	-3.2E-13
-13.8	-41.75	-1.4E-13	-3.8	-3.53	-1.2E-14	6.2	-8.75	-2.5E-14	16.2	-57.39	-4.5E-13
-13.7	-41.16	-1.4E-13	-3.7	-3.37	-1.5E-14	6.3	-9.02	-2E-14	16.3	-58.09	-5.5E-13
-13.6	-40.56	-1.7E-13	-3.6	-3.21	-1.8E-15	6.4	-9.29	-1.4E-14	16.4	-58.80	-6.8E-13
-13.5	-39.97	-1.7E-13	-3.5	-3.06	-4.9E-15	6.5	-9.57	-7.1E-15	16.5	-59.52	-7.1E-14
-13.4	-39.39	-1.8E-13	-3.4	-2.91	-9.8E-15	6.6	-9.86	-3.6E-15	16.6	-60.24	-1.8E-13
-13.3	-38.81	-1.9E-13	-3.3	-2.76	-1.2E-14	6.7	-10.15	-2.7E-14	16.7	-60.96	-3.1E-13
-13.2	-38.23	-2.1E-13	-3.2	-2.62	-1.4E-14	6.8	-10.44	-2E-14	16.8	-61.69	-4.2E-13
-13.1	-37.66	-2.3E-13	-3.1	-2.49	-1.8E-15	6.9	-10.74	-1.4E-14	16.9	-62.42	-5.2E-13
-13	-37.10	-2.3E-13	-3	-2.35	-4.9E-15	7	-11.04	-5.3E-15	17	-63.15	-6.5E-13
-12.9	-36.53	-2.4E-13	-2.9	-2.23	-8E-15	7.1	-11.35	0	17.1	-63.90	-2.8E-14
-12.8	-35.98	-2.5E-13	-2.8	-2.10	-1E-14	7.2	-11.66	-2.3E-14	17.2	-64.64	-1.7E-13
-12.7	-35.42	-2.5E-13	-2.7	-1.98	-2.4E-15	7.3	-11.97	-1.8E-14	17.3	-65.39	-2.7E-13
-12.6	-34.87	-2.7E-13	-2.6	-1.87	-4.2E-15	7.4	-12.29	-1.1E-14	17.4	-66.14	-3.8E-13

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-12.5	-34.33	-2.8E-13	-2.5	-1.76	-6E-15	7.5	-12.61	-7.1E-15	17.5	-66.90	-5.1E-13
-12.4	-33.79	-2.8E-13	-2.4	-1.65	-7.1E-15	7.6	-12.94	-3.2E-14	17.6	-67.66	-6.4E-13
-12.3	-33.25	-3E-13	-2.3	-1.55	-9.5E-15	7.7	-13.27	-2.5E-14	17.7	-68.43	-7.7E-13
-12.2	-32.72	-3.1E-13	-2.2	-1.45	-8.9E-16	7.8	-13.61	-1.8E-14	17.8	-69.20	-1.1E-13
-12.1	-32.19	-3.2E-13	-2.1	-1.36	-2.4E-15	7.9	-13.95	-1.2E-14	17.9	-69.98	-2.1E-13
-12	-31.67	-3.3E-13	-2	-1.27	-4.7E-15	8	-14.30	0	18	-70.75	-3.6E-13
-11.9	-31.15	-3.3E-13	-1.9	-1.18	-4.2E-15	8.1	-14.65	-3.2E-14	18.1	-71.54	-4.7E-13
-11.8	-30.63	-3.4E-13	-1.8	-1.10	-3.6E-15	8.2	-15.00	-2.5E-14	18.2	-72.33	-6E-13
-11.7	-30.12	-3.4E-13	-1.7	-1.03	-3.1E-15	8.3	-15.36	-1.2E-14	18.3	-73.12	-7.2E-13
-11.6	-29.62	-3.6E-13	-1.6	-0.95	-4.7E-15	8.4	-15.72	-7.1E-15	18.4	-73.92	-7.1E-14
-11.5	-29.12	-3.7E-13	-1.5	-0.89	-2.9E-15	8.5	-16.09	0	18.5	-74.72	-1.7E-13
-11.4	-28.62	-3.7E-13	-1.4	-0.82	-3.2E-15	8.6	-16.46	-2.8E-14	18.6	-75.52	-3.1E-13
-11.3	-28.13	-3.8E-13	-1.3	-0.77	-2.2E-15	8.7	-16.83	-2.1E-14	18.7	-76.33	-4.3E-13
-11.2	-27.64	-3.8E-13	-1.2	-0.71	-4.2E-15	8.8	-17.21	-1.8E-14	18.8	-77.15	-5.8E-13
-11.1	-27.15	-3.9E-13	-1.1	-0.66	-3.3E-15	8.9	-17.60	-7.1E-15	18.9	-77.97	-7.1E-13
-11	-26.67	-4E-13	-1	-0.62	-8.9E-16	9	-17.99	0	19	-78.79	-1.4E-14

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-10.9	-26.20	-4.1E-13	-0.9	-0.57	-3.3E-16	9.1	-18.38	-2.8E-14	19.1	-79.62	-1.6E-13
-10.8	-25.73	-4.1E-13	-0.8	-0.54	-8.9E-16	9.2	-18.78	-2.1E-14	19.2	-80.45	-2.7E-13
-10.7	-25.26	-4.1E-13	-0.7	-0.51	-1.1E-15	9.3	-19.18	-1.8E-14	19.3	-81.28	-4E-13
-10.6	-24.80	-4.3E-13	-0.6	-0.48	-1.3E-15	9.4	-19.59	0	19.4	-82.12	-5.4E-13
-10.5	-24.34	-4.3E-13	-0.5	-0.45	-5.6E-17	9.5	-20.00	-3.9E-14	19.5	-82.97	-6.5E-13
-10.4	-23.89	-4.3E-13	-0.4	-0.43	-6.1E-16	9.6	-20.41	-2.8E-14	19.6	-83.82	-8.2E-13
-10.3	-23.44	-4.4E-13	-0.3	-0.42	-5.6E-16	9.7	-20.83	-2.5E-14	19.7	-84.67	-8.5E-14
-10.2	-22.99	0	-0.2	-0.41	-5E-16	9.8	-21.25	-7.1E-15	19.8	-85.53	-2.1E-13
-10.1	-22.55	-7.1E-15	-0.1	-0.40	-8.3E-16	9.9	-21.68	0	19.9	-86.39	-3.6E-13
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ST	Rmi	©2020 Ins	stitute of Er	vironmental	Science and Resear	ch Limited P	age 17 of 141				

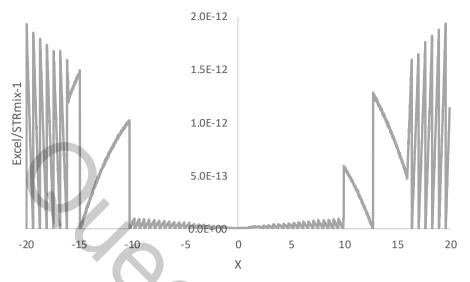


Figure 3.1 Comparison of normal density STRmix™ and MS Excel

## 3.3. Cumulative Normal approximation

A cumulative normal approximation is used in the calculation of the probability of dropout. Within STRmix<sup>TM</sup> V2.8, a cumulative normal approximation function via the Normal Distribution from the Apache Commons Math library is implemented. This is slow, therefore a hybrid approach is applied where if X < -7, the Apache method is used in order to obtain a cumulative probability and a programmed approximation is used otherwise. The value of -7<sup>1</sup> is implemented as it appears to be the best balance of speed (as most X values will be greater than this) and accuracy.

In Table 3.2 we give a comparison of the implemented method and the Excel norm.s.dist function. This defines how similar the repeat calculation should be when a cumulative normal distribution probability is used.

The maximum absolute difference was 1.64E-02 and the minimum 0.

# Table 3.2 Comparison of log probabilities using STRmix<sup>™</sup> and the standard normal distribution in MS Excel

	Apache [-∞,-7] then		
x	programmed approximation (-7,∞]	Excel	Difference
-20	-88.5600953	-88.5600953	0.00E+00
-19.5	-84.2604976	-84.2604976	0.00E+00
-19	-80.0691923	-80.0691923	0.00E+00

<sup>&</sup>lt;sup>1</sup> This setting is controlled within the default.ini file by the CnaSwitchOverPoint parameter (default -7). Note that this parameter should not be less than -8, otherwise a –Infinity/NaN error will arise.



		Dal	e of Issue: 29 September 202
-18.5	-75.9861647	-75.9861647	0.00E+00
-18	-72.0113987	-72.0113987	0.00E+00
-17.5	-68.1448772	-68.1448772	0.00E+00
-17	-64.3865815	-64.3865815	0.00E+00
-16.5	-60.7364910	-60.7364910	0.00E+00
-16	-57.1945838	-57.1945838	1.21E-13
-15.5	-53.7608355	-53.7608355	1.07E-13
-15	-50.4352196	-50.4352196	1.21E-13
-14.5	-47.2177070	-47.2177070	0.00E+00
-14	-44.1082654	-44.1082654	0.00E+00
-13.5	-41.1068595	-41.1068595	0.00E+00
-13	-38.2134498	-38.2134498	0.00E+00
-12.5	-35.4279927	-35.4279927	0.00E+00
-12	-32.7504392	-32.7504392	6.39E-14
-11.5	-30.1807344	-30.1807344	4.62E-14
-11	-27.7188167	-27.7188167	4.26E-14
-10.5	-25.3646162	-25.3646162	0.00E+00
-10	-23.1180534	-23.1180534	8.17E-14
-9.9	-22.6816496	-22.6816496	0.00E+00
-9.8	-22.2495470	-22.2495470	6.04E-14
-9.7	-21.8217446	-21.8217446	3.20E-14
-9.6	-21.3982418	-21.3982418	3.91E-14
-9.5	-20.9790376	-20.9790376	8.88E-14
-9.4	-20.5641312	-20.5641312	0.00E+00
-9.3	-20.1535216	-20.1535216	3.20E-14
-9.2	-19.7472079	-19.7472079	4.26E-14
-9.1	-19.3451892	-19.3451892	0.00E+00
-9	-18.9474644	-18.9474644	9.24E-14
-8.9	-18.5540326	-18.5540326	9.59E-14
-8.8	-18.1648926	-18.1648926	4.26E-14
-8.7	-17.7800435	-17.7800435	0.00E+00
-8.6	-17.3994840	-17.3994840	7.46E-14
-8.5	-17.0232130	-17.0232130	7.11E-14
-8.4	-16.6512293	-16.6512293	9.95E-14
-8.3	-16.2835317	-16.2835317	0.00E+00
-8.2	-15.9201190	-15.9201190	5.51E-14
-8.1	-15.5609897	-15.5609897	9.41E-14
-8	-15.2061426	-15.2061426	5.68E-14
-7.9	-14.8555761	-14.8555761	2.13E-14
-7.8	-14.5092890	-14.5092890	2.13E-14
-7.7	-14.1672796	-14.1672796	9.95E-14
-7.6	-13.8295465	-13.8295465	7.28E-14
-7.5	-13.4960879	-13.4960879	9.06E-14
-7.4	-13.1669023	-13.1669023	7.82E-14
-7.3	-12.8419879	-12.8419879	4.09E-14
-7.2	-12.5213429	-12.5213429	0.00E+00



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	2.000-00
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	2.40E-03
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-6.4 -10.1076326 -10.1096434	2.01E-03
-6.3 -9.8254399 -9.8273305	1.89E-03
-6.2 -9.5474908 -9.5492648	1.77E-03
-6.1 -9.2737821 -9.2754437	1.66E-03
-6 -9.0043113 -9.0058643	1.55E-03
-5.9 -8.7390753 -8.7405237	1.45E-03
-5.8 -8.4780710 -8.4794187	1.35E-03
-5.7 -8.2212951 -8.2225463	1.25E-03
-5.6 -7.9687444 -7.9699029	1.16E-03
-5.5 -7.7204152 -7.7214850	1.07E-03
-5.4 -7.4763040 -7.4772892	9.85E-04
-5.3 -7.2364068 -7.2373114	9.05E-04
-5.2 -7.0007198 -7.0015477	8.28E-04
-5.1 -6.7692387 -6.7699939	7.55E-04
	6.86E-04
	6.22E-04
-4.8 -6.0999866 -6.1005471	5.60E-04
-4.7 -5.8852837 -5.8857870	5.03E-04
-4.6 -5.6747629 -5.6752126	4.50E-04
-4.5 -5.4684185 -5.4688184	4.00E-04
-4.4 -5.2662449 -5.2665986	3.54E-04
-4.3 -5.0682360 -5.0685469	3.11E-04
-4.2 -4.8743855 -4.8746570	2.72E-04
-4.1 -4.6846865 -4.6849221	2.36E-04
-4 -4.4991321 -4.4993349	2.03E-04
-3.9 -4.3177148 -4.3178879	1.73E-04
-3.8 -4.1404268 -4.1405732	1.46E-04
-3.7 -3.9672598 -3.9673823	1.23E-04
-3.6 -3.7982051 -3.7983064	1.01E-04
-3.5 -3.6332533 -3.6333360	8.27E-05
-3.4 -3.4723948 -3.4724613	6.65E-05
	5.25E-05
	4.05E-05
	3.05E-05
	2.23E-05
-2.9 -2.7291163 -2.7291318	1.56E-05
-2.8 -2.5925767 -2.5925869	1.03E-05
	6.17E-06
	3.15E-06
	3.15E-06



		Dat	e of Issue: 29 September 20
-2.5	-2.2069308	-2.2069318	1.02E-06
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-2.1	-1.7480126	-1.7480111	-1.54E-06
-2	-1.6430174	-1.6430161	-1.34E-06
-1.9	-1.5418686	-1.5418676	-1.01E-06
-1.8	-1.4445396	-1.4445389	-6.48E-07
-1.7	-1.3510019	-1.3510016	-3.02E-07
-1.6	-1.2612251	-1.2612251	-1.83E-08
-1.5	-1.1751765	-1.1751767	1.78E-07
-1.4	-1.0928214	-1.0928217	2.79E-07
-1.3	-1.0141222	-1.0141225	2.91E-07
-1.2	-0.9390389	-0.9390391	2.32E-07
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1.4	-0.0365695	-0.0365695	-2.45E-08
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1.8	-0.0158916	-0.0158916	2.42E-08
1.9	-0.0126540	-0.0126540	2.99E-08



2.1       -7.82853E-03       -7.82856E-03       2.14E-08         2.2       -6.08054E-03       4.68256E-03       2.14E-08         2.3       -4.68256E-03       4.68258E-03       3.09E-09         2.4       -3.57481E-03       -2.70524E-03       -6.35E-09         2.6       -2.02908E-03       -2.02906E-03       -1.47E-08         2.7       -1.50833E-03       -1.50830E-03       -2.15E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3.1       -4.20458E-04       -4.20428E-04       -2.91E-08         3.1       -4.20458E-04       -2.09699E-04       -2.36E-08         3.2       -2.96551E-04       -2.98523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.24E-08         3.4       -1.46374E-04       -1.0141E-04       -1.92E-08         3.6       -6.91216E-05       -6.8145E-05       -1.161E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -6.827105       -6.42E-09         4	[		Dat	e of Issue: 29 September 202
2.3       -4.68256E-03       -4.68258E-03       1.27E-08         2.4       -3.57481E-03       -3.57482E-03       3.09E-09         2.5       -2.70524E-03       -2.70528E-03       -6.35E-09         2.6       -2.02908E-03       -2.12908E-03       -2.15E-08         2.7       -1.50833E-03       -1.11110E-03       -2.63E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.811069E-04       -2.91E-08         3.1       -4.20458E-04       -4.20428E-04       -2.96E-08         3.2       -2.98551E-04       -2.88523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46331E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -2.88685E-05       +8.33E-09         4       -1.37613E-05       -1.37649E-05       +3.04215-09         4.1       -3.77645E-06       -5.79602E-06       -3.63E-09         4.2       -5.79965E-06       -1.37549E-07       -9.51E-10	2.1	-7.82853E-03	-7.82856E-03	2.79E-08
2.4       -3.57481E-03       -3.57482E-03       3.09E-09         2.5       -2.70524E-03       -2.0290E-03       -1.47E-08         2.7       -1.50833E-03       -1.50830E-03       -2.15E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3       -5.86679E-04       -5.86649E-04       -3.01E-08         3.1       -4.20458E-04       -4.20428E-04       -2.99E-08         3.2       -2.98551E-04       -2.98523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46374E-04       -2.94E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -2.08885E-05       -8.33E-09         4.1       -1.37613E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -2.08865E-06       -3.63E-09         4.1       -3.71151E-06       -3.7632E-06       -3.63E-09         4.2       -5.79605E-06       -1.3759E-06       -1.916E-09         <	2.2	-6.08054E-03	-6.08056E-03	2.14E-08
2.5       -2.70524E-03       -2.70523E-03       -6.35E-09         2.6       -2.02906E-03       -1.47E-08         2.7       -1.50833E-03       -1.50830E-03       -2.15E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3.1       -4.20458E-04       -4.20428E-04       -2.96E-08         3.2       -2.98551E-04       -2.98523E-04       -2.246E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.31055E-05       -1.32E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -1.37549E-05       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -2.66E-09         4.4       -2.35256E-06       -2.35064E-06       -1.36E-09         4.4	2.3	-4.68256E-03	-4.68258E-03	1.27E-08
2.6       -2.02908E-03       -1.47E-08         2.7       -1.50833E-03       -2.15E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -3.01E-08         3       -5.86679E-04       -5.86649E-04       -3.01E-08         3.1       -4.20458E-04       -2.96E-08       -2.96E-08         3.2       -2.98551E-04       -2.09999E-04       -2.54E-08         3.3       -2.10025E-04       -2.09999E-04       -2.24E-08         3.5       -1.01061E-04       -1.46374E-04       -1.46374E-04         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -4.86194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -4.87E-09         4.2       -5.79965E-06       -5.79602E-06       -3.63E-09         4.3       -3.71151E-06       -3.70885E-06       -2.66E-09         4.4       -2.35256E-06       -2.35064E-06       -1.91E-09         4.4       -2.35256E-06       -2.35064E-06       -1.91E-09         4.5       -1.44683E-07       -1.	2.4	-3.57481E-03	-3.57482E-03	3.09E-09
2.7       -1.50833E-03       -1.50830E-03       -2.15E-08         2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3       -5.66679E-04       -5.86649E-04       -3.01E-08         3.1       -4.20458E-04       -2.99523E-04       -2.96E-08         3.2       -2.98551E-04       -2.99999E-04       -2.54E-08         3.3       -2.10025E-04       -2.09999E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -2.0885E-05       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -1.36E-09         4.2       -5.79965E-06       -5.79602E-06       -3.63E-09         4.3       -3.1151E-06       -3.70885E-06       -1.36E-09         4.4       -2.35256E-06       -2.35064E-06       -1.36E-09         4.4       -2.35256E-06       -1.47559E-06       -1.36E-10	2.5	-2.70524E-03	-2.70523E-03	-6.35E-09
2.8       -1.11113E-03       -1.11110E-03       -2.63E-08         2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3       -5.86679E-04       -5.86649E-04       -2.91E-08         3.1       -4.20458E-04       -4.20428E-04       -2.96E-08         3.2       -2.89551E-04       -2.98523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01061E-05       -6.81055E-05       -1.61E-08         3.6       -6.91216E-05       -6.81045E-05       -1.06E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -8.33E-09       -4         4.1       -1.37613E-05       -1.37549E-05       -6.42E-09         4.1       -3.7151E-06       -3.70885E-06       -2.66E-09         4.4       -2.3556E-06       -2.370885E-06       -1.36E-09         4.5       -1.47695E-06       -1.47559E-06       -1.36E-09         4.5       -1.47695E-06       -1.37549E-07       -2.66E-19         4.5 </td <td>2.6</td> <td>-2.02908E-03</td> <td>-2.02906E-03</td> <td>-1.47E-08</td>	2.6	-2.02908E-03	-2.02906E-03	-1.47E-08
2.9       -8.11098E-04       -8.11069E-04       -2.91E-08         3       -5.86679E-04       -5.86649E-04       -3.01E-08         3.1       -4.20458E-04       -2.96E08         3.2       2.98551E-04       -2.99523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -8.33E-09       -4.487E-09         4.1       -8.97640E-06       -8.97153E-06       -4.682-09         4.1       -8.7960E-06       -3.70885E-06       -2.366E-09         4.3       -3.71151E-06       -3.70885E-06       -1.36E-09         4.4       -2.3526E-06       -2.35064E-06       -1.36E-09         4.5       -1.47695E-06       -1.4759E-06       -1.36E-09         4.6       -9.18379E-07       -9.17428E-07       -9.51E-10         4.7       -5.65589E	2.7	-1.50833E-03	-1.50830E-03	-2.15E-08
3         -5.86679E-04         -5.86649E-04         -3.01E-08           3.1         -4.20428E-04         -2.96E-08           3.2         -2.98551E-04         -2.98523E-04         -2.79E-08           3.3         -2.10025E-04         -2.09999E-04         -2.54E-08           3.4         -1.46374E-04         -1.46351E-04         -2.24E-08           3.5         -1.01061E-04         -1.01041E-04         -1.92E-08           3.6         -6.91216E-05         -6.91055E-05         -1.61E-08           3.7         -4.68326E-05         -8.91055E-05         -1.06E-08           3.9         -2.08968E-05         -2.08868E-05         -8.33E-09           4         -1.37613E-05         -1.37549E-05         -6.42E-09           4.1         -8.97640E-06         -8.97153E-06         -2.66E-09           4.3         -3.71151E-06         -3.70885E-06         -2.66E-09           4.4         -2.35256E-06         -2.35064E-06         -1.91E-09           4.5         -1.47695E-06         -1.47559E-06         -1.36E-09           4.5         -1.47695E-07         -2.08107E-07         -2.98E-10           5         -1.24688E-07         -1.24491E-07         -4.45E-10           4.9         -2.081	2.8	-1.11113E-03	-1.11110E-03	-2.63E-08
3.1       -4.20458E-04       -4.20428E-04       -2.96E-08         3.2       2.98551E-04       -2.98523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -2.08885E-05       -8.33E-09         4       -1.37613E-05       -1.37549E-05       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -4.87E-09         4.2       -5.79965E-06       -5.79602E-06       -3.63E-09         4.3       -3.71151E-06       -3.70885E-06       -1.36E-09         4.4       -2.35256E-06       -2.35064E-06       -1.91E-09         4.4       -2.35256E-06       -2.45094E-07       -6.55E-10         4.5       -1.47695E-07       -5.64934E-07       -6.55E-10         4.5       -1.4688E-07       -1.24491E-07       -1.97E-10         5.1       -7.38	2.9	-8.11098E-04	-8.11069E-04	-2.91E-08
3.2       -2.98551E-04       -2.98523E-04       -2.79E-08         3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.8194E-05       -1.61E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -2.08885E-05       -8.33E-09         4       -1.37613E-05       -1.37549E-06       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -2.66E-09         4.1       -8.97640E-06       -3.70885E-06       -2.66E-09         4.2       -5.79602E-06       -2.35064E-06       -1.91E-09         4.3       -3.71151E-06       -3.70885E-06       -1.36E-09         4.4       -2.35256E-06       -2.35064E-06       -1.91E-09         4.5       -1.47695E-06       -1.47559E-06       -1.36E-09         4.6       -9.18379E-07       -9.51E-10       -1.47559E-10         4.7       -5.6589E-07       -2.08107E-07       -2.98E-10	3	-5.86679E-04	-5.86649E-04	-3.01E-08
3.3       -2.10025E-04       -2.09999E-04       -2.54E-08         3.4       -1.46374E-04       -1.46351E-04       -2.24E-08         3.5       -1.01061E-04       -1.01041E-04       -1.92E-08         3.6       -6.91216E-05       -6.91055E-05       -1.61E-08         3.7       -4.68326E-05       -4.68194E-05       -1.32E-08         3.8       -3.14321E-05       -3.14215E-05       -1.06E-08         3.9       -2.08968E-05       -2.08885E-05       -6.42E-09         4       -1.37613E-05       -1.37549E-06       -6.42E-09         4.1       -8.97640E-06       -8.97153E-06       -4.687E-09         4.2       -5.79965E-06       -5.79602E-06       -3.66E-09         4.3       -3.71151E-06       -3.7085E-06       -1.91E-09         4.5       -1.47695E-06       -1.47559E-06       -1.36E-09         4.5       -1.47695E-06       -1.47559E-06       -1.36E-09         4.6       -9.18379E-07       -9.17428E-07       -9.51E-10         4.7       -5.65589E-07       -5.64934E-07       -4.45E-10         5.1       -7.28832E-08       -1.226810       -1.226810         5.2       -4.33575E-08       -4.32750E-08       -3.29E-11         <	3.1	-4.20458E-04	-4.20428E-04	-2.96E-08
3.4 $-1.46374E-04$ $-1.46351E-04$ $-2.24E-08$ $3.5$ $-1.01061E-04$ $-1.01041E-04$ $-1.92E-08$ $3.6$ $-6.91126E-05$ $-6.91055E-05$ $-1.61E-08$ $3.7$ $-4.68326E-05$ $-4.68194E-05$ $-1.32E-08$ $3.8$ $-3.1421E-05$ $-3.14215E-05$ $-1.06E-08$ $3.9$ $-2.08968E-05$ $-2.08885E-05$ $-8.33E-09$ $4$ $-1.37613E-05$ $-1.37549E-05$ $-6.42E-09$ $4.1$ $-8.97640E-06$ $-8.97153E-06$ $-3.63E-09$ $4.2$ $-5.79965E-06$ $-5.79602E-06$ $-3.63E-09$ $4.3$ $-3.71151E-06$ $-3.70885E-06$ $-2.66E-09$ $4.4$ $-2.35256E-06$ $-2.35064E-06$ $-1.91E-09$ $4.5$ $-1.47695E-06$ $-1.4759E-06$ $-1.36E-09$ $4.6$ $-9.18379E-07$ $-9.17428E-07$ $-9.51E-10$ $4.7$ $-5.65589E-07$ $-5.64934E-07$ $-4.45E+10$ $4.9$ $-2.08405E-07$ $-2.08107E-07$ $-2.98E-10$ $5$ $-1.24688E-07$ $-1.24491E-07$ $-1.97E-10$ $5.1$ $-7.38332E-08$ $-7.37548E-08$ $-1.28E-11$ $5.2$ $-4.33575E-08$ $-4.32750E-08$ $-5.24E-11$ $5.4$ $-1.45038E-08$ $-1.24401E-09$ $-1.24E-11$ $5.4$ $-1.45038E-08$ $-1.24E-12$ $-2.60159E-09$ $-7.51E-12$ $5.8$ $-7.91970E-10$ $-7.89334E-10$ $-2.64E-12$ $6$ $-4.30004E-10$ $-2.30232E-11$ $-3.329E-11$ $5.7$ $-2.60099E-09$ $-2.64E-12$	3.2	-2.98551E-04	-2.98523E-04	-2.79E-08
3.5 $-1.01061E-04$ $-1.01041E-04$ $-1.92E-08$ $3.6$ $-6.91216E-05$ $-6.91055E-05$ $-1.61E-08$ $3.7$ $-4.68326E-05$ $-4.68194E-05$ $-1.32E-08$ $3.8$ $-3.14221E-05$ $-3.14215E-05$ $-1.06E-08$ $3.9$ $-2.08968E-05$ $-2.08885E-05$ $-6.42E-09$ $4$ $-1.37613E-05$ $-1.37549E-05$ $-6.42E-09$ $4.1$ $-8.97640E-06$ $-8.97153E-06$ $-4.87E-09$ $4.2$ $-5.79965E-06$ $-5.79602E-06$ $-3.63E-09$ $4.3$ $-3.71151E-06$ $-3.70885E-06$ $-2.66E-09$ $4.4$ $-2.35256E-06$ $-2.35064E-06$ $-1.91E-09$ $4.5$ $-1.47695E-06$ $-1.47559E-06$ $-1.36E-09$ $4.6$ $-9.18379E-07$ $-9.17428E-07$ $-9.55E-10$ $4.7$ $-5.65589E-07$ $-5.64934E-07$ $-6.55E-10$ $4.8$ $-3.44983E-07$ $-3.44538E-07$ $-4.45E-10$ $4.9$ $-2.08405E-07$ $-2.08107E-07$ $-2.98E-10$ $5$ $-1.24688E-07$ $-1.24491E-07$ $-1.97E-10$ $5.1$ $-7.38832E-08$ $-7.37548E-08$ $-1.28E-10$ $5.2$ $-4.33575E-08$ $-4.32750E-08$ $-8.267+11$ $5.3$ $-2.51987E-08$ $-2.51462E-08$ $-5.24E-11$ $5.4$ $-4.66702E-09$ $-8.24706E-09$ $-2.03E-11$ $5.6$ $-8.26740E-09$ $-8.24706E-09$ $-2.03E-11$ $5.6$ $-4.66702E-09$ $-4.65459E-09$ $-1.24E+11$ $5.7$ $-2.09099E-09$ $-2.60159E-09$ $-7.51E-12$ <td>3.3</td> <td>-2.10025E-04</td> <td>-2.09999E-04</td> <td>-2.54E-08</td>	3.3	-2.10025E-04	-2.09999E-04	-2.54E-08
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5.8-1.44449E-09-1.44001E-09-4.48E-125.9-7.91970E-10-7.89334E-10-2.64E-126-4.30004E-10-4.28470E-10-1.53E-126.1-2.31208E-10-2.30325E-10-8.83E-136.2-1.23110E-10-1.22608E-10-5.02E-136.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
5.9-7.91970E-10-7.89334E-10-2.64E-126-4.30004E-10-4.28470E-10-1.53E-126.1-2.31208E-10-2.30325E-10-8.83E-136.2-1.23110E-10-1.22608E-10-5.02E-136.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6-4.30004E-10-4.28470E-10-1.53E-126.1-2.31208E-10-2.30325E-10-8.83E-136.2-1.23110E-10-1.22608E-10-5.02E-136.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6.1-2.31208E-10-2.30325E-10-8.83E-136.2-1.23110E-10-1.22608E-10-5.02E-136.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6.2-1.23110E-10-1.22608E-10-5.02E-136.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6.3-6.49149E-11-6.46329E-11-2.82E-136.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6.4-3.38963E-11-3.37397E-11-1.57E-136.5-1.75272E-11-1.74413E-11-8.60E-14				
6.5 -1.75272E-11 -1.74413E-11 -8.60E-14				
0.0  -0.07400L-12  -4.00L-14				
	0.0	-0.37+00L-12	-0.320101-12	-7.UUL-14



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-4.55081E-12	-4.52578E-12	-2.50E-14
-2.28507E-12	-2.27176E-12	-1.33E-14
-1.13617E-12	-1.12923E-12	-6.94E-15
-5.59406E-13	-5.55838E-13	-3.57E-15
-2.72760E-13	-2.70928E-13	-1.83E-15
-1.31679E-13	-1.30763E-13	-9.16E-16
-6.29706E-14	-6.24884E-14	-4.82E-16
-2.97977E-14	-2.95566E-14	-2.41E-16
-1.39827E-14	-1.38381E-14	-1.45E-16
-6.50921E-15	-6.41278E-15	-9.64E-17
-2.98942E-15	-2.94120E-15	-4.82E-17
-1.35006E-15	-1.35006E-15	3.94E-30
-6.26813E-16	-6.26813E-16	0.00E+00
-2.89298E-16	-2.89298E-16	0.00E+00
	-2.28507E-12 -1.13617E-12 -5.59406E-13 -2.72760E-13 -1.31679E-13 -6.29706E-14 -2.97977E-14 -1.39827E-14 -6.50921E-15 -2.98942E-15 -1.35006E-15 -6.26813E-16	-2.28507E-12       -2.27176E-12         -1.13617E-12       -1.12923E-12         -5.59406E-13       -5.55838E-13         -2.72760E-13       -2.70928E-13         -1.31679E-13       -1.30763E-13         -6.29706E-14       -6.24884E-14         -2.97977E-14       -2.95566E-14         -1.39827E-14       -1.38381E-14         -6.50921E-15       -6.41278E-15         -2.98942E-15       -2.94120E-15         -1.35006E-15       -1.35006E-15         -6.26813E-16       -6.26813E-16

#### 3.4. Unit test exponential function

Code

Random randomNumberGenerator = new Random(12345);

System.out.println("Exponential unit test: generating 10000 random exponential densities with lambda chosen between 0.0001 and 10 and x-value between -5 and 2 (on log scale))");

System.out.println("lambda,xval,density");

for (int i = 0; i < 10000; i++) {

double lambda = (10-0.0001)\*randomNumberGenerator.nextDouble()+0.0001;

double xval = 7\*randomNumberGenerator.nextDouble()-5;

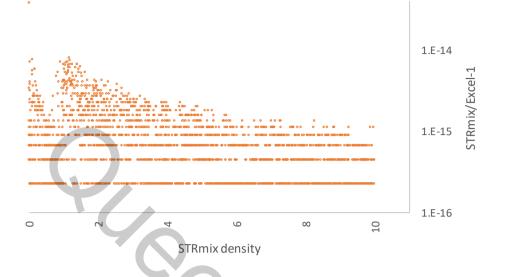
double density = STRMath.exponentialProbability(xval, lambda);

```
System.out.println(lambda + "," + xval + "," + density);
```

```
}
```

Exponential unit test: generating 10000 random exponential densities with lambda chosen between 0.0001 and 10 and x-value between -5 and 2 (on log scale). Figure 3.2 is a comparison of the exponential density for STRmix<sup>™</sup> and MS Excel.





#### Figure 3.2 Comparison of exponential density STRmix<sup>™</sup> and MS Excel

Examination of Figure 3.2 suggests that we should expect precision to 14 sig fig most of the time.

#### 3.5. Unit test Gamma distribution

Code

System.out.println("Gamma unit test: generating 10000 random gamma densities with alpha nd beta chosen between 0.1 and 10 and x-value between 0 and 200");

System.out.println("alpha,beta,xval,density");

for (int i = 0; i < 10000; i++) {

double alpha = (10-0.1)\*randomNumberGenerator.nextDouble()+0.1;

double beta = (10-0.1)\*randomNumberGenerator.nextDouble()+0.1;

GammaParameters gammaPair = new GammaParameters(alpha, beta);

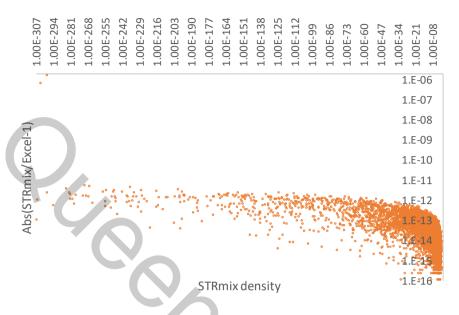
double xval = 200\*randomNumberGenerator.nextDouble();

double Log10density = STRMath.gammaPenalty(xval,gammaPair);

```
System.out.println(alpha + "," + beta + "," + xval + "," + Math.pow(10,Log10density));
```

}

Gamma unit test: generating 10000 random gamma densities with alpha and beta chosen between 0.1 and 10 and x-value between 0 and 200. Figure 3.3 is a comparison of the gamma density for STRmix and MS Excel.



## Figure 3.3 Comparison of gamma density STRmix<sup>™</sup> and MS Excel

Examination of Figure 3.3 suggests that we should expect precision to 11 sig fig most of the time.

#### 3.6. Variables with stochastic elements

When a stochastic element is present in the calculation the acceptance criterion must be calculated using sampling theory. This includes the random walk, and HPD processes.



#### 4. RESULTS OF DEVELOPMENTAL VALIDATION

#### 4.1. Pre checks for missing stutter peaks

Acceptance criterion: Peaks flagged as missing when dropout penalty less than threshold. Expected peak height in error message rounded to 0 dp. Precision expected to 0 dp for each.

Stutter peaks were artificially deleted from input files to simulate dropout. Within Excel, the dropout probability for back and forward stutter peaks were calculated using the following formula:

=LOG(NORM.DIST(LOG( $Z/E_{a-1}$ ),0,SQRT( $k^2/(1000/O_a+O_a)$ ),1)) back stutter

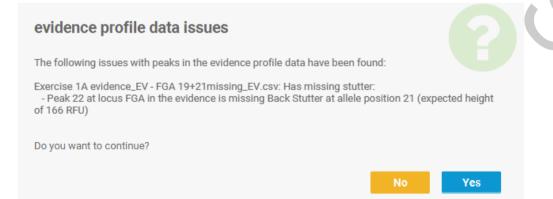
=LOG(NORM.DIST(LOG( $Z/E_{a+1}$ ),0,SQRT( $k^2/(1000/E_{a+1}+E_{a+1})$ ),1)) forward stutter

Where back stutter variance is proportional to parent peak height and forward stutter variance is proportional to the expected height of the forward stutter, z is the analytical threshold,  $E_{a-1}$  the expected height of the back stutter peak,  $E_{a+1}$  the expected height of the forward stutter peak,  $O_a$  the observed height of the presumed parent peak and  $k^2$  the mode of the relevant stutter variance prior.

Where the dropout penalty was less than the threshold of -8 a warning was given. An example is given below for a single source GlobalFiler<sup>™</sup> profile.

Locus	Stutter	Allele	O <sub>a</sub>	E <sub>a-1</sub>	Mode of k <sup>2</sup>	Probability of dropout
FGA	19	20	2538	136	6.7944	-6.59179
FGA	21	22	2482	166	6.7944	-10.6692

As expected, a message was given for the missing 21 stutter peak (and not for the 19 peak).



These tests satisfy the acceptance criteria.



# 4.2. Modelling drop-in peaks in stutter positions

Acceptance criterion: stutter penalties are written to an extended output (labelled in 'part peak designations') and are rounded to 4 dp. Drop-in penalties are written to the same file also rounded to 4 dp. Precision is expected to 4 dp.

A back stutter penalty is assigned for one example by the equation:

# LOG(NORM.DIST(LOG(O/E),0,SQRT(*k*<sup>2</sup>/(1000/O<sub>17.3</sub>+O<sub>17.3</sub>)),0))

Peak	0	E	O <sub>16</sub>	Drop-in penalty STRmix	Drop-in penalty Excel	Stutter penalty STRmix	Stutter penalty Excel
15	1552	386.4461	5174	-3.8028	-3.8028	-59.2468	-59.2468

From the peak designation file:

> expected stutter height (386.4461) and < drop-in cap (2000) and <= stutter max (1,552.2) but drop-in penalty (-3.8028) > stutter balance penalty (-59.2468).

These tests satisfy the acceptance criteria.

#### 4.3. Smart start

Acceptance criterion: Smart start is rounded to 0 dp within the STRmix<sup>™</sup> extended output and log. Precision expected to 0 dp.

The values necessary to replicate smart start are written to a text file as part of the extended output from STRmix<sup>™</sup>. The smart start algorithm was checked in Excel for 17 different interpretations including; multiple numbers of contributors, assumed contributors, replicate PCRs, multi-kit analysis, drop-in, and combinations of each. A summary of the results from STRmix<sup>™</sup> and Excel<sup>™</sup> for a number of profiles is given in the table below.

Sample	STRmix™	Excel
Single source	4695	4695
Two person with assumed contributor	17403; 404	17403; 404
Three person with two assumed contributors	256; 987; 50	256; 987; 50
Three person multi kit	662; 225; 50	662; 225; 50
Two person multi kit with replicates	295; 50	295; 50
	295, 50	295, 50

The values are rounded up to the nearest whole number. These tests satisfy the acceptance criteria.

#### 4.4. Expected peak heights given mass parameters

In STRmix<sup>TM</sup>, the amount of DNA for each contributor is modelled by an exponential equation using mass parameters proposed at each MCMC iteration. The total allelic product (TAP) of allele *a* at locus *l* for replicate number *y*, kit *k* is:

$$T_{anyk}^{l} = A_{k}^{l} R_{y} t_{n} B_{k} \times e^{-d_{n} \times \left(m_{ak}^{l} - \min\left(m_{ak}\right)\right)} X_{an}^{l}$$

The amplification efficiency ( $A_k^l$ ), replicate efficiency ( $R_y$ ), kit efficiency ( $B_k$ ), template DNA ( $t_n$ ), and slope ( $d_n$ ), are collectively referred to as the mass parameters. The allele molecular weight ( $m_a^l$ ) and the minimum molecular weight across the profile are taken from the input file. Molecular weight values are truncated. The expected height of an allele is calculated from the TAP using the following general formula (considering back and forward stutter only):

$$E_a = \frac{T_{anyk}^l}{1 + SR_a + FSR_a}$$

Per allele stutter ratios are determined from the stutter files. If a value is present in the stutter exceptions file, this is used as the per allele stutter ratio otherwise a value is calculated from the stutter regression file. A minimum stutter ratio of 0.001 is applied to all stutter types and all loci where the stutter is enabled.

Expected stutter peaks heights are calculated using the following general formula (considering back and forward stutter only):

$$E_{a-1} = SR_aO_a$$
$$E_{a+1} = FSR_aO_a$$

#### Where:

 $E_{a-1}$  and  $E_a$  are the expected heights of the stutter and allele respectively, and  $O_a$  the observed height of allele, *a*.  $SR_a$  and  $FSR_a$  are the stutter and forward stutter ratios for the allele, respectively.

Note that total allelic product is not output within STRmix<sup>™</sup> extended outputs.

#### 4.4.1. Expected peak heights using stutter regression file

Acceptance criterion: Expected allele height is output to 15 sig fig and stutter height to 5 dp in STRmix<sup>™</sup>. Precision is expected to 10 sig fig for expected allele height and 5 dp for stutter peak height.

Expected allele and stutter peak heights were calculated in Excel for selected loci from 14 different interpretations including; single source profiles, mixed DNA profiles, assumed contributors, replicate PCRs, multi-kit analysis. The extended output for a single source profile was analysed and the expected height of various peaks using a stutter regression file is given below.



A summary of the genotype combination at one locus and the mass parameters for five iterations is provided in the following table:

Genotype	<i>t</i> <sub>1</sub>		$A^l$
9,9	4297.75181663863	0.000677705280678254	0.862441558791565
9,9	4326.12830893682	0.000594011278317268	0.854594074489788
9,9	4308.13636117358	0.000600868570364307	0.862632285566135
9,9	4204.82077296148	0.000566925558660158	0.859444700195893
9,9	4219.69644998446	0.000544719478243954	0.839947111416278

The observed peak heights for the double back stutter, back stutter, allele, and forward stutter from the STRmix<sup>™</sup> extended output, and their expected height calculated in Excel are in the following table:

E <sub>a-2</sub>	E <sub>a-1</sub>	E <sub>a</sub> STRmix	E <sub>a+1</sub>	E <sub>a-2</sub> Excel	E <sub>a-1</sub> Excel	E <sub>a</sub> Excel	E <sub>a+1</sub> Excel
STRmix	STRmix	0'	STRmix				
32.82240	227.56864	6397.771771	39.59202	32.82240	227.56864	6397.771771	39.59202
32.82240	227.56864	6464.196892	39.59202	32.82240	227.56864	6464.196892	39.59202
32.82240	227.56864	6491.003262	39.59202	32.82240	227.56864	6491.003262	39.59202
32.82240	227.56864	6345.009245	39.59202	32.82240	227.56864	6345.009245	39.59202
32.82240	227.56864	6244.320076	39.59202	32.82240	227.56864	6244.320076	39.59202



Where  $O_9 = 6838$  rfu, total allelic product =  $t_1 * A^{l*} \exp(-d_1 * (243-89))*2$  and where the truncated size of the 9 allele from the input file was 243 bp and the minimum allele size in the input file was 89 bp.

 $E_9 = TAP/(1+SR_9+FSR_9+DBSR_9)$ 

From the back stutter regression file:

Locus	Intercept	Slope
D16S539	-0.05042	0.0093
$SR_{0} = -0.05$	$042+0.0093^{\circ}$	*9 = 0.03328

From the forward stutter regression file:

Locus	Intercept	Slope
D16S539	0.00579	0

 $FSR_9 = 0.00579 + 0*9 = 0.00579$ 

From the double back stutter regression file

Locus	Intercept	Slope
D16S539	0.0048	0
	10.0*0 0.0	0.4.0

FSR9 = 0.0048+0\*9 = 0.0048

These tests satisfy the acceptance criteria.



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Page 31 of 141

# 4.4.2. Expected peak heights using stutter exceptions file

Acceptance criterion: Expected allele height is output to 15 sig fig and stutter height to 5 dp fig in STRmix<sup>™</sup>. Precision is checked to 10 sig fig for expected allele height and 5 dp for stutter peak height.

Expected allele and stutter peak heights were calculated in Excel for selected loci from 14 different interpretations including; single source profiles, mixed DNA profiles, assumed contributors, replicate PCRs, multi-kit analysis. The extended output for a single source profile was analysed and the expected height of various peaks using a stutter exceptions file was calculated.

A summary of the genotype combination at one locus for five iterations and the mass parameters is provided in the following table:

Genotype			A <sup>l</sup>
16,17.3	4297.75181663863	0.000677705280678254	1.47854746472871
16,17.3	4326.12830893682	0.000594011278317268	1.48990557727800
16,17.3	4308.13636117358	0.000600868570364307	1.50722565299489
16,17.3	4204.82077296148	0.000566925558660158	1.50312903983602
16,17.3	4219.69644998446	0.000544719478243954	1.51072359454794

The observed peak heights for the double back stutter, back stutter (using exceptions file), two base pair stutter, allele, and forward stutter from the STRmix<sup>™</sup> extended output for 17.3 allele, and their expected height calculated in Excel are in the following table:

				· · · · · · · · · · · · · · · · · · ·		
	Iteration	Ea-2	E <sub>a-1</sub>	Ea-0.2	Ea	E <sub>a+1</sub>
	STRmix	40.35028	353.78048	90.80216	5412.441936	35.075
1	Excel	40.35028	353.78048	90.80216	5412.441936	35.075
	STRmix	40.35028	353.78048	90.80216	5538.952747	35.075
2	Excel	40.35028	353.78048	90.80216	5538.952747	35.075
	STRmix	40.35028	353.78048	90.80216	5575.984523	35.075
3	Excel	40.35028	353.78048	90.80216	5575.984523	35.075
	STRmix	40.35028	353.78048	90.80216	5447.035046	35.075
4	Excel	40.35028	353.78048	90.80216	5447.035046	35.075
	STRmix	40.35028	353.78048	90.80216	5506.870953	35.075
5	Excel	40.35028	353.78048	90.80216	5506.870953	35.075

These tests satisfy the acceptance criteria.

#### 4.5. Log(likelihood) values

#### 4.5.1. Taylor quantum effect, no dropout

Acceptance criterion: log(likelihood) is output to 15 sig fig in STRmix<sup>™</sup>. Precision is expected in Excel to 14 sig fig.

Log(likelihood) values were calculated in Excel for selected loci from 20 different profiles including; single source profiles, mixed DNA profiles, assumed contributors, replicate PCRs, multi-kit analysis.

The peak designation, expected and observed peak heights and log(likelihood), or log(p), for each peak for each locus may be recovered from the extended output. They are provided in the following format:

|--|

The number of columns per peak and the column titles will depend on the number of stutter types modelled and their labels. The log(likelihood) for a range of peak types and for five iterations is replicated below, where b=1000. Note that the log(likelihood) is only calculated where O>z or if O = 0 rfu, where E > 1/2z.

Log(likelihood) values for back stutter peaks (where back stutter variance is proportional to observed allele height)

Log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT( $k^2/(1000/O_{17.3}+O_{17.3})),0)$ ), where  $O_{17.3} = 5612$  rfu

Peak	0	E STRmix	$k_{a-1}^2$	log(p) STRmix	log(p) Excel
16.3	389	353.78048	4.44207231525283	0.6856397209	0.6856397209
16.3	389	353.78048	4.41008369263630	0.6838286641	0.6838286641
16.3	389	353.78048	4.44204914141806	0.6856384224	0.6856384224
16.3	389	353.78048	4.50591579181502	0.6891442771	0.6891442771
16.3	389	353.78048	4.62206500969138	0.6951631890	0.6951631890

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

Log(likelihood) values for alleles

 $Log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT(c^{2}/(1000/E+E)),0))$ 

Peak	0	E	<b>C</b> <sup>2</sup>	log(p) STRmix	log(p) Excel
17.3	5612	5412.44193661015	15.1477137911941	0.8582554553	0.8582554553
17.3	5612	5538.95274770251	15.1353361900925	0.8800620020	0.8800620020
17.3	5612	5575.98452399238	15.1408686264031	0.8833771962	0.8833771962
17.3	5612	5447.03504672663	15.1157058294766	0.8661448485	0.8661448485
17.3	5612	5506.87095368646	15.0870371828278	0.8767214594	0.8767214594

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

Log(likelihood) values for forward stutter peaks (where forward stutter variance is inversely proportional to expected height)

Peak	0	E	$k_{a+1}^2$	log(p) STRmix	log(p) Excel
18.3	160	35.075	7.29505631979703	-0.7511961539	-0.7511961539
18.3	160	35.075	7.33617167573560	-0.7478081824	-0.7478081824
18.3	160	35.075	7.26994719134881	-0.7532874358	-0.7532874358
18.3	160	35.075	7.25507448069025	-0.7545341997	-0.7545341997
18.3	160	35.075	7.31469120565082	-0.7495726006	-0.7495726006

# Log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT( $k_{aul}^2/(1000/E+E)),0)$ )

To obtain the expected precision we need to convert the normal to the standard normal. The

standard deviation is  $\sqrt{\frac{k_{a+1}^2}{1000/O_a + O_a}} \approx 0.18$ . After multiplying the densities by the sd the

difference between log(p) STRmix and log(p) Excel is less than 2 x 10<sup>-16</sup>.

The precision for a log density at about these X values is about 7 or 8 x 10<sup>-15</sup>. Hence this passes the acceptance criterion.

#### 4.5.2. Taylor quantum effect, peaks below z (dropped peaks)

Acceptance criterion: log(likelihood) is output to 15 sig fig in STRmix™. Precision is expected in Excel depending on the magnitude of X as discussed in Section 3.3 Cumulative Normal approximation.

The log(likelihood) for a range of peak types and for five iterations is replicated below, where *b*=1000. Note that the log(likelihood) is only calculated where O > z or if O = 0 rfu, where E > z1/2*z*.

#### Expected peak heights of dropped alleles

 $E = t_1 * A^l * EXP(-d_1 * (180-89))$  where the average size of the peaks present within the input file at this locus = 180 bp and the minimum size of all peaks = 89 bp.

<i>t</i> <sub>1</sub>		$A^l$	E STRmix	E Excel
37.1018888544871	0.000393288488845183	1.140624695068760	41.77363718	41.77363718

36.7548324832931	0.000366787940226078	1.133357982788490	41.15521305	41.15521305
38.1070866793617	0.000306483714594276	1.131144032422140	42.67084256	42.67084256
34.2766578008051	0.000351912621406127	1.138931875598040	38.58803859	38.58803859
36.4061185511573	0.000369398065685203	1.116414797954950	40.15187720	40.15187720

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

## Log(likelihood) values for dropped alleles

Log(p)=LOG(NORM.DIST(LOG(75/E),0,SQRT(c2/(1000/E+E)),1)), where z=75 rfu and E is the expected height of the peak as calculated above

E	X	C <sup>2</sup>	log(p) STRmix	log(p) Excel
41.7736371828539	0.550316	14.0162763290725	-0.1493852071	-0.1493852269
41.1552130576283	0.562488	14.0532298013709	-0.1468445153	-0.1468445384
42.6708425609767	0.531052	14.0621270978156	-0.1534719551	-0.1534719692
38.5880385905585	0.619481	14.0004125872925	-0.1353700746	-0.1353701099
40.1518772040832	0.586299	13.9358788054211	-0.1419666585	-0.1419666874

X is calculated in Excel using the formula:

=-LOG10(E/75)/SQRT(k2/(1000/Oa+Oa)) (for stutter peaks with missing data where Z=75 rfu and b=1000)

There are differences of  $3.5 \times 10^{-8}$ . At X values for cumulative probabilities in this region the STRmix<sup>TM</sup> log cumulative probability is within about  $3.18 \times 10^{-8}$  and  $4.24 \times 10^{-8}$  of the Excel one. Hence these differences are within the acceptance criterion.

# Log(likelihood) values for dropped two base pair stutter peaks (where double back stutter variance is proportional to its expected height)

Log(p)=LOG(NORM.DIST(LOG(75/E),0,SQRT( $k_{a-0.2}^2/(1000/E+E))$ ,1)), where z=75 rfu, E is the expected height of the stutter peak.

Peak	0	E	Х	$k_{a-0.2}^2$	log(p) STRmix	log(p) Excel
17.1	0	90.80216	-0.785	1.13840043346852	-0.6652500046	-0.6652498681

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17.1	0	90.80216	-0.782	1.14810583375804	-0.6632931004	-0.6632929633
17.1	0	90.80216	-0.784	1.14195938321898	-0.6645290834	-0.6645289467
17.1	0	90.80216	-0.775	1.16922560903803	-0.6591316384	-0.6591315002
17.1	0	90.80216	-0.779	1.15584515134752	-0.6617529342	-0.6617527966

There are differences of about 1.38 x 10<sup>-7</sup>. It is expected that values for cumulative probabilities for log(p) from STRmix<sup>TM</sup> are about 2.3 x 10<sup>-9</sup> to  $^{-2}$ .8 x 10<sup>-7</sup> of the Excel values given the precision of the expected stutter. This is within the expected precision.

## Log(likelihood) values for dropped double back stutter peaks (where double back stutter variance is inversely proportional to expected height)

Log(p)=LOG(NORM.DIST(LOG(75/E),0,SQRT( $k_{a-2}^2/(1000/E+E))$ ,1)), where z = 75 rfu.

Peak	0	E	Х	$k_{a-2}^2$	log(p) STRmix	log(p) Excel
15.3	0	40.35028	1.01459	4.58585705891935	-0.07322092095	-0.07322091516
15.3	0	40.35028	1.01214	4.60804357284515	-0.07352113438	-0.07352112906
15.3	0	40.35028	1.00985	4.62895253220254	-0.07380294564	-0.07380294077
15.3	0	40.35028	1.00529	4.67106866110201	-0.07436732801	-0.07436732405
15.3	0	40.35028	1.01361	4.59469771834702	-0.07334069346	-0.07334068785

There are differences of about 5.8 x  $10^{-9}$ . It is expected that values for cumulative probabilities for log(p) from STRmix<sup>TM</sup> are about 1.7 x  $10^{-8}$  of the Excel one given the precision of the expected stutter. This is within the expected precision. These tests satisfy the acceptance criteria.

#### 4.6. Probabilities and expected stutter peak heights given saturated alleles

Acceptance criterion: log(likelihood) is output to 15 sig fig in STRmix<sup>™</sup>. Precision is checked in Excel to 10 sig fig. Expected back stutter peak height is output to 5 dp and precision is expected to 5 dp in STRmix<sup>™</sup>. For saturated alleles, expected back stutter height is output to 15 sig fig and precision is checked to 10 sig fig.

The log(likelihood) and expected peak heights for two profiles with observed and expected saturated alleles were replicated in Excel from the extended output.

The following tables replicate one profile analysed on a CE 3500 that was interpreted with the saturation setting artificially low at s=5000. The log(likelihood) and expected peak heights for a range peak types for five iterations is replicated below.

#### Log(likelihood) for alleles when $O_a < s$

When E<s,  $log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT(c^{2}/(1000/E+E)),0))$ 

When  $E \ge s$ ,  $log(p)=LOG(NORM.DIST(LOG(O/5000),0,SQRT(c^2/(1000/5000+5000)),0))$ 

Allele	0	E	<i>C</i> <sup>2</sup>	log(p) STRmix	log(p) Excel
16	4614	5457.97190970947	14.5973718712686	0.7777079518	0.7777079518
16	4614	5450.60722423450	14.6979162814592	0.7768368887	0.7768368887
16	4614	5450.65600358263	14.6112390254063	0.7775877118	0.7775877118
16	4614	5400.36513172157	14.6263675177696	0.7774565720	0.7774565720
16	4614	5470.21487602310	14.6079473365208	0.7776162506	0.7776162506

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### Expected height and log(likelihood) values for back stutter peaks when Oa < s

Log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT( $k_{a-1}^2/(1000/O_{16}+O_{16})),0)$ )

Where  $O_{16} = 4614$  rfu and  $SR_{16} = 0.09351$ .

Peak	0	E STRmix	$k_{a-1}^2$	log(p) STRmix	E Excel	log(p) Excel
15	460	431.45514	7.92737048732550	0.8855569253	431.45514	0.8855569253
15	460	431.45514	7.81140930154187	0.8873043970	431.45514	0.8873043970
15	460	431.45514	7.75784407225460	0.8881130083	431.45514	0.8881130083
15	460	431.45514	7.95319255985181	0.8851684089	431.45514	0.8851684089
15	460	431.45514	7.90429767203593	0.8859042695	431.45514	0.8859042695

There are no differences in the 10<sup>th</sup> significant figure for log(p) values. The expected heights are the same to 5 dp. Hence these pass the acceptance criterion.

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 37 of 141

#### Log(likelihood) values for alleles when Oa > s

When E<s,  $log(p)=LOG(NORM.DIST(LOG(O/E),0,SQRT(c^2/(1000/E+E)),0))$ 

When  $E \ge s$ ,  $log(p)=LOG(NORM.DIST(LOG(O/5000),0,SQRT(c^2/(1000/5000+5000)),0))$ 

Peak	0	E	<i>C</i> <sup>2</sup>	log(p) STRmix	log(p) Excel
15	5079	5046.552683849510	14.7168046299834	0.8630772453	0.8630772453
15	5079	5022.356145983540	14.7071510373387	0.8632174866	0.8632174866
15	5079	5005.710029338570	14.7465562768069	0.8626456007	0.8626456007
15	5079	4901.811784821820	14.7310133251947	0.8447959226	0.8447959226
15	5079	4913.943178152170	14.7515704348481	0.8473230332	0.8473230332

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### Expected height of stutter peaks when Oa>s

If  $O_a > s$ ,  $E_{a-1} = SR_a * E_a$ , otherwise,  $E_{a-1} = SR_a * O_a$ 

Peak	O <sub>16</sub>	E <sub>16</sub> STRmix	E <sub>16.3</sub> STRmix	E <sub>16.3</sub> Excel	
15	5174	5731.45453209985	426.0989833	426.0989833	
15	5174	5757.93526666577	428.0768294	428.0768294	
15	5174	5733.92140341930	426.2832340	426.2832340	
15	5174	5652.13940162810	420.1749363	420.1749363	$\mathbf{O}$
15	5174	5705.61608453316	424.1691097	424.1691097	9,0
Whore S	<u> </u>	07460	•	u	

Where  $SR_{16} = 0.07469$ 

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### Log(likelihood) values of stutter peaks when Oa > s

=LOG(NORM.DIST(LOG(O/E),0,SQRT( $k_{a-1}^2/(1000/E_{17.3}+E_{17.3})),0)$ )

Peak	0	E	$k_{a-1}^2$	log(p) STRmix	log(p) Excel

				2410 01 1004	0. 20 Optombol 2020
15	400	426.098983397538	7.92737048732550	0.9117190326	0.9117190326
15	400	428.076829462266	7.81140930154187	0.8953842479	0.8953842479
15	400	426.283234016387	7.75784407225460	0.9122292728	0.9122292728
15	400	420.174936302603	7.95319255985181	0.9555829672	0.9555829672
15	400	424.169109748782	7.90429767203593	0.9278379128	0.9278379128

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.7. Shifted log normal for composite peaks

Acceptance criterion: log(likelihood) is output to 15 sig fig in STRmix<sup>™</sup>. Precision is expected in Excel to 13 sig fig and less for dropped alleles depending on the magnitude of X as discussed in Section 3.3 Cumulative Normal approximation. Precision if checked to 10 sig fig.

The log(likelihood) using the shifted log normal (SLN) model from the post burn-in extended output was replicated in Excel for 17 profiles containing composite alleles, back, forward, double back, and exotic stutter types such as 2 bp stutter at SE33.

In the following tables, the log(likelihood) is replicated for a composite allele and forward stutter peak

а	l	Oa	Ea	E <sub>a+1</sub>	<b>c</b> <sup>2</sup>	<i>k</i> <sup>2</sup>
1	2	3687	3745.84	39.22497	15.29999120742390	7.42248582823617

Allelic distribution				
$\mu^i_{{}_{\log_{10}}}$	$\mu^i_{ m ln}$	$\mathrm{var}^i_{\mathrm{log}_{10}}$	var <sub>ln</sub>	
3.57354886354320	8.22840034228039	0.00408424105865942	0.02165422995164470	

Forward stutter distribution					
$\mu^i_{\log_{10}}$	$\mu^i_{ m ln}$	$\operatorname{var}^i_{\log_{10}}$	var <sub>ln</sub> <sup>i</sup>		
1.59356262010246	3.66931353380045	0.114688009996105	0.608064143492875		

Moments

Allele	Double back stutter
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Date of Issue: 29 September 2020

$E[E^i]$	$VAR[E^i]$	$E[E^i]$	$VAR[E^i]$
3786.613856853140	313874.060228181000	53.162093040873	2365.174551699180

Combined	
E[E]	VAR[E]
3839.7759498940	316239.23477988

Shifted LN approximation parameters					
τ μ	$\sigma^2$	Pr(O E,SLN)			
1540.98091900597 7.7110798499248	0.0581210301903956	0.000760708718226549			

log(p)	
STRmix™	Excel
0.810107220	0.810107220

The log(likelihood) calculation using the SLN model for one composite back and double back stutter peak for one post burn-in iteration when the observed peak height is 0 is given below.

а	Oa	O <sub>a+1</sub>	E <sub>a-1</sub>	E <sub>a-2</sub>	$k_{a-1}^2$	$k_{a-2}^2$
13	0	2840	173.2968	26.537	47.0454136111740	5.09159310850694

Back stutter distribution			
$\mu^i_{\log_{10}}$	$\mu^i_{ m ln}$	$\mathrm{var}^i_{\mathrm{log}_{10}}$	var <sup>i</sup> <sub>ln</sub>
2.23879054335343	5.15500573146164	0.0165632329179250	0.0878165733109604

Double back stutter dis	stribution		
$\mu^i_{\log_{10}}$	$\mu^i_{ m ln}$	$\operatorname{var}^i_{\log_{10}}$	var <sub>ln</sub>
1.42385182444069	3.27853998558951	0.0792833151420747	0.2974635355438830

#### Moments

Back stutter		Double back stutter	
$E[E^i]$ $VAR[E^i]$		$E[E^i]$ $VAR[E^i]$	
181.0754903582340	3009.5703031133600	30.7925184609700	328.4865144150490



Combined	
E[E]	VAR[E]
211.8680088192040	3338.0568175284100

Sh	Shifted LN approximation parameters					
τ	$\tau$ $\mu$ $\sigma^2$ $Pr(O E,SLN)$					
0		5.3200990346478700	0.0717288924378393	0.0000907138508489173		

log(p)	
STRmix™	Excel
-4.042193828	-4.042326396

These tests satisfy the acceptance criteria.

#### 4.8. Expected peak heights for replicate profiles

Acceptance criterion: Expected allele height is output to 15 sig fig and stutter height to 5 dp in STRmix<sup>™</sup>. Precision is checked to 10 sig fig for expected allele height and 5 dp for stutter peak height.

The expected peak heights for four replicate interpretations were calculated in Excel from the extended output.

In the following tables, the expected peak heights for peaks from two inputs from replicate PCR of the same extract interpreted assuming two contributors, assuming the presence of one individual are given. The extended output for each peak within a replicate analysis is in the form:

Peak	0	E	E Back	E Forward	log(p)	
------	---	---	--------	-----------	--------	--

The number of columns per peak and the column titles will depend on the number of stutter types modelled and their labels.

At CSF1PO, the following alleles were present in the input files:

REP1	Allele	Height	Size

S.

CSF1PO	12	42	100

REP2	Allele	Height	Size
CSF1PO	12	30	306

Note the sizes are truncated to an integer (whole base pairs). The sizes for the 12 allele were deliberately edited to be disparate to demonstrate that an average is used within the calculation of total allelic product. The truncated average size for the 12 allele = 203 bp. The truncated minimum size across both replicates = 50 bp. The reference at CSF1PO for the assumed contributor is 12,12.



In the following tables the masss parameters for the CSF1PO locus is reproduced:

C1	C2	<i>t</i> <sub>1</sub>	$t_2$	$d_1$	$d_2$
12.0,12.0	-1.0,-1.0	69.1948660741329	15.4207059682330	0.00275215129535	0.00320287534391
12.0,12.0	-1.0,-1.0	65.1320439369875	13.4396100559091	0.00279456705228	0.00323987532790
12.0,12.0	-1.0,-1.0	71.9197613892744	13.9764345793742	0.00271672687702	0.00331881538199
12.0,12.0	-1.0,-1.0	67.8653376376408	14.0764588358648	0.00271771011286	0.00326439928352
12.0,12.0	-1.0,-1.0	67.7397706397475	13.8723750374247	0.00275929288135	0.00320878099001

A <sup>CSF</sup>	R <sub>1</sub>	R <sub>2</sub>
0.905006756472633	1.074484034783070	0.930679254068109
0.899379226505035	1.069603337934040	0.934926027747367
0.902102967100105	1.060018817984670	0.943379478773043
0.894086759179492	1.058915258208140	0.944362631710640
0.893622128364227	1.067144803732010	0.937079950633505

The total allelic product was calculated using the formula  $T_{any}^{l} = A_{k}^{l} t_{n} R_{y} \times e^{-d_{n} \times (m_{a}^{l} - \min(m_{a}))} X_{an}^{l}$ 

Total allelic product =  $t_1 * A^{CSF} * R_1 * EXP(-d_1 * (203-50))*2$ 

 $E=TAP/(1+SR_{12}+FSR_{12})$ , where  $SR_{12} = 0.06182$  and  $FSR_{12} = 0.00691$ 

# **STRmix**.

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Page 43 of 141

The expected peak height for the 12 allele for contributor 1, replicate 1, is given in the table below:

Peak	0	E STRmix	TAP Excel	E Excel
12	42	82.64475683	88.3249309684726	82.64475683
12	42	76.45951720	81.7145798276304	76.45951720
12	42	84.93005732	90.7673001620110	84.93005732
12	42	79.33540506	84.7881274535081	79.33540506
12	42	79.25672041	84.7040348128612	79.25672041

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

The expected back and forward stutter peaks for the 12 allele for contributor 1, replicate 1, is given in the table below:

Stutter	E STRmix™	E Excel
Back	2.59644	2.59644
Forward	0.29022	0.29022

These numbers are the same to 5 dp. Hence these tests pass the acceptance criterion.

The expected peak height for the -1 (dropped allele) for contributor 2, replicate 1, is given in the table below  $E = t_2 * A^{CSF} * R_1 * EXP(-d_2 * (203-50))$ , where the average size for the 12 allele = 203 bp and the minimum size across both replicates = 50 bp.

	-		
Peak	0	E STRmix	E Excel
-1	0	16.96818105	16.96818105
-1	0	17.52566261	17.52566261
	Ŭ	11102000201	11102000201
-1	0	17.79693489	17.79693489
	U	11.1 0000-00	11.10000400
-1	0	17.30576420	17.30576420
	0	17.30370420	17.30370420
-1	0	10.00560000	10.00560020
-	0	18.98568938	18.98568938

There are no differences in the 10th significant figure. These tests satisfy the acceptance criteria.

## 4.8.1. Replicates conditioning on an individual whose alleles has 'dropped out' of the profile

Acceptance criterion: Expected allele height is output to 15 sig fig in STRmix<sup>™</sup>. Precision is checked to 10 sig fig.

Replicate analysis was undertaken for a profile where the assumed contributor was dropping out of the profile. Note the size of the 15 allele was deliberately edited to 50 bp.

REP1	Allele	Height	Size
D22S1045	15	67	50

The assumed contributor is a 15,17 at this locus. There is no profiling information at D22S1045 for replicate 2 and no 17 allele present in replicate 1. From the size regression file:

Locus	Intercept	Slope
D22S1045	64.66242	2.986364

The expected size of a 17 allele at D22S1045 is:

64.66242+17\*2.986364=115 bp

The truncated average of the 15 and 17 alleles = 82 bp. The minimum size across both replicates is 50 bp. In the following tables the mass parameter information for the D22S1045 locus is reproduced:

15.0,17.0         -1.0,17.0         62.6064927040726         24.6900162420881         0.00274189043810030         0.00327577388864725           15.0,17.0         -1.0,17.0         59.9694063736518         28.9075195936333         0.00275795102315941         0.00334134956135675           15.0,17.0         -1.0,17.0         56.2602821011490         28.5733034489033         0.00272091403365052         0.00345935157950035						
15.0,17.0         -1.0,17.0         62.6064927040726         24.6900162420881         0.00274189043810030         0.00327577388864725           15.0,17.0         -1.0,17.0         59.9694063736518         28.9075195936333         0.00275795102315941         0.00334134956135675           15.0,17.0         -1.0,17.0         56.2602821011490         28.5733034489033         0.00272091403365052         0.00345935157950035	C1	C2	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>	$d_1$	$d_2$
15.0,17.0         -1.0,17.0         59.9694063736518         28.9075195936333         0.00275795102315941         0.00334134956135675           15.0,17.0         -1.0,17.0         56.2602821011490         28.5733034489033         0.00272091403365052         0.00345935157950035	15.0,17.0	-1.0,17.0	61.0049782822864	24.2206638791544	0.00279051611587667	0.00334788500742973
15.0,17.0         -1.0,17.0         56.2602821011490         28.5733034489033         0.00272091403365052         0.00345935157950035	15.0,17.0	-1.0,17.0	62.6064927040726	24.6900162420881	0.00274189043810030	0.00327577388864725
	15.0,17.0	-1.0,17.0	59.9694063736518	28.9075195936333	0.00275795102315941	0.00334134956135675
15.0,17.0         -1.0,17.0         55.8464923952199         33.7996322498695         0.00263403638606275         0.00336544763984819	15.0,17.0	-1.0,17.0	56.2602821011490	28.5733034489033	0.00272091403365052	0.00345935157950035
	15.0,17.0	-1.0,17.0	55.8464923952199	33.7996322498695	0.00263403638606275	0.00336544763984819

$A^{CSF}$	R <sub>1</sub>	R <sub>2</sub>
1.194852671427460	0.970139090556726	1.030780029105040
1.209568325940900	0.946377388364466	1.056660918038420
1.207952666474240	0.931028149369584	1.074081380543770
1.215290276466970	0.931011251799601	1.074100874792910
1.224243289216440	0.948867930345718	1.053887446312630

Total allelic product 17 allele C1 = $t_1 * A^{D22} * R_1 * EXP(-d_1 * (115-50))$ 

 $E_{17} C1 = TAP C1/(1+SR_{17}+FSR_{17})$ 

Where  $SR_{17} = 0.10167$  and  $FSR_{17} = 0.04783$ 

The expected peak height for the 17 allele for replicate 1, is given in the table below:

<b>—</b> ·	-				
Peak	0	E STRmix	TAP C1	TAP C2	E Excel
			Ť		
17	0	70.96136195	58.9847531506033	22.5853324167460	70.96136195
17	0	72.03966455	59.9670828938541	22.8425115175653	72.03966455
	_				
17	0	71.80420316	56.3751732355576	26.1637582998248	71.80420316
	Ũ		001010102000010		
17	0	68.86123500	53.3369025942188	25.8190870424085	68.86123500
1.,	Ŭ	00.00120000	00.0000020012100	20.0100010121000	00.00120000
17	0	75.00144286	54.6654360283063	31.5487225497124	75.00144286
1 ''	Ŭ	70.00144200	04.0004000200000	01.0407220407124	10.00144200
L			u		

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

Total allelic product -1 allele C2 = $t_2 * A^{D22} * R_2 * EXP(-d_1 * (50-50))$ , where 50 is the average of the observed peak sizes within the input file that represent a Q (or -1 or dropped allele) and also the minimum observed size across both replicates.

The expected peak height for the -1 (dropped allele) for contributor 2, replicate 2, is given in the table below:

Peak	0	E STRmix	E Excel
-1	0	29.83090282	29.83090282
-1	0	31.55639809	31.55639809
-1	0	37.50575683	37.50575683
-1	0	37.29800019	37.29800019
-1	0	43.60878014	43.60878014

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.9. Summation of probabilities

Acceptance criterion: Expected log(likelihood) is output to 15 sig fig. Precision is checked to 10 sig fig.

The expected peak heights for 15 interpretations from post burn-in extended output were summed in Excel. Profiles included single source and mixed, replicates, multikits, drop-in peaks, saturated peaks, assumed contributors, Mx priors.

The probability densities (allele and stutter penalties, drop-in penalties, individual allele and stutter log(p) values, LSAE variance probabilites and penalties) were extracted from the extended output and summed. These values were compared to the total MCMC probability, also within the extended output.

A summary of the typical results for four iterations is provided in the table below.

Iteration	1	2	3	4
Allele variance penalty	- 1.34077076296007 00	- 1.33631425116962 00	- 1.33927846208580 00	- 1.33827552823015 00
Back stutter variance penalty	- 1.29871296222762 00	- 1.29840255024794 00	- 1.29866690154001 00	- 1.29794585526275 00
Forward stutter variance penalty	- 1.20188736662765 00	- 1.20154542895622 00	- 1.20237914652235 00	- 1.20182192478225 00



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1	1	1		sue. 29 September 2020
Locus amplificati on probability	7.49071058240757 00	7.54177670857820 00	7.38839716049472 00	7.51269352004503 00
LSAE variance penalty	1.37926336558872 00	1.43992490019691 00	1.38166010649079 00	1.41811931570741 00
log(p)	0.07765119666248	0.09408212414902	0.07448382854508	0.08871414334155
	86	30	70	45
log(p)	0.10460753982867	0.09109484872367	0.10978377513736	0.10091704534599
	40	11	30	00
log(p)	0.02288363745907 45	- 0.00422974361952 70	0.03706313568885 59	0.00859110888586 39
log(p)	-	-	-	-
	0.15595878913957	0.14362469696833	0.17295537694231	0.15551714183698
	80	20	60	40
log(p)	0.09005978295879	0.08796467229924	0.09026206901440	0.09009034687048
	22	57	70	62
log(p)	0.07089371810830	0.04139280571356	0.07116386799664	0.06289241841584
	23	13	73	68
log(p)	0.07134053091824	0.07218647858705	0.07016119283862	0.07259727155959
	03	41	12	52
log(p)	-	-	-	-
	0.13045372876405	0.11223514131668	0.13933322686329	0.12545291131422
	20	80	40	70
log(p)	0.07310905623483	0.07139572379547	0.07221945652781	0.07291348702832
	70	03	12	90
log(p)	-	-	-	-
	0.12182581250582	0.10451335105480	0.13030137271674	0.11716298188754
	30	20	10	30
log(p)	-	-	-	-
	0.08506645541962	0.03724394398297	0.10486545908755	0.05833329889822
	70	55	10	63
log(p)	-	-	-	-
	0.22139711116884	0.16513281992565	0.27247806803098	0.21073096795626
	10	50	90	80

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log(p)	07	85	23	
log(p)	0.07747099447832	0.03059821641537	0.07602212508782	- 0.15206647148170 70
log(p)				0.01860394098945 40
log(p)	- 0.14860674233691 50	- 0.13329228436798 00	- 0.16692452579670 70	Se
log(p)	0.02596768307428 68	- 0.00676223087679 40	0.04242073459286 12	
log(p)		0.08500113120786 12	0.11660597092151 30	0.10799997210181 40
log(p)		0.09273137654571 70	0.07432052132024 13	0.08257332031682 14
log(p)	0.10999345653731 40	- 0.19779708651690 90	- 0.23861005759511 40	- 0.22307417129874 60
log(p)	0.07475499715858 41	- 0.29394202514304 60	- 0.22397050397992 00	- 0.25487361890591 60
log(p)	- 0.29875422867669 90			
log(p)	- 0.10882760109532 60			
log(p)	- 0.28306625528020 10	- 0.20857844917477 60	- 0.35054201873236 80	- 0.27735414633258 60
log(p)	- 0.10646176659628 80	- 0.04292974142073 18	- 0.13201190848891 90	sue: 29 September 2020 - 0.07670160993350 90



Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

1	1	i		sue: 29 September 2020
log(p)	- 0.24922415688701 50	- 0.21547660830435 80	- 0.25981939597602 60	
log(p)				- 0.01519502577905 26
log(p)	),			- 0.19125396660901 80
log(p)	0.13516037413101	0.12381529737256	0.13765399090586	0.12902618236899
	50	30	20	60
log(p)	0.08382583590053	0.08783065166344	0.07370520849060	0.08462013132238
	77	39	49	28
log(p)	0.10916602563201	0.08999498056497	0.11663835304406	0.09799987475572
	90	91	00	83
log(p)	0.04526014131507	0.05365973143303	0.03084789023565	0.04628307083650
	80	19	58	47
log(p)	0.09269916223906	0.10301100647766	0.09073670088024	0.10052080641569
	62	40	67	70
log(p)	0.11356553331930	0.09235699159225	0.12210662932244	0.10678617548455
	90	98	40	10
log(p)	-	-	-	-
	0.07270577171778	0.11019563764639	0.04419143025009	0.08441683716886
	49	90	85	53
log(p)	0.02277479962162 09	0.04062403195113 16	- 0.00914345420001 12	0.01863131128395 09
log(p)	0.17714430058783	0.15772283306184	0.17744333484571	0.16808076787808
	70	20	40	60
log(p)	-	-	-	-
	0.26223217118125	0.31876713181226	0.23690079580695	0.27194169354807
	50	90	60	80
log(p)	-	-	-	-
	0.48529685953508	0.53708861378550	0.46606097664770	0.49254057930389
	10	10	10	70



Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

			1 1
0.00273654287962 67	- 0.02350388204873 61	0.02245980124360 51	0.00928164897170 71
-	-	-	-
0.17393615108603	0.21839545031865	0.15872809711909	0.18084225671815
40	60	00	50
-	-	-	-
0.07940120194502	0.11197369587214	0.05354981829472	0.07043789896141
12	40	76	30
-	-	-	-
1.03183917829908	1.09059119724857	1.00608705433918	1.04421374513849
00	00	00	00
- 0.00820782277198 99	- 0.05069255626170 69	0.02368044821871 45	- 0.00260525038507 03
0.12664963690540	0.12507146048425	0.12454597279344	0.12635190873343
70	90	00	10
-	-	-	-
0.26598590474049	0.24263015065217	0.29298602326214	0.26654490448658
90	20	40	90
-	-	-	-
0.23714968203082	0.21236461456583	0.25041448471267	0.22730069998735
30	10	00	60
-	-	-	-
0.18686089047348	0.17006217915709	0.20634157716074	0.18809152909207
20	80	60	90
0.17219149393516	0.17657033213745	0.17186828346236	0.17556023286864
30	50	80	40
0.17921172310023	0.16039014130058	0.18390998635335	0.17625066417474
50	50	70	30
0.01897561395463	0.04266323191053	0.00602509104443	0.03457359900108
92	39	76	14
0.09857794395215	0.07766347327584	0.11565002863923	0.09400808593142
51	54	80	70
0.02925310186986	0.01202442225298	0.03923861908990	0.03283253348291
76	31	97	89
	67 - 0.17393615108603 40 - 0.07940120194502 12 - 0.03183917829908 0.12664963690540 70 - 0.26598590474049 90 - 0.26598590474049 90 - 0.26598590474049 90 - 0.23714968203082 30 - 0.18686089047348 20 - 0.17219149393516 30 - 0.01897561395463 92 - 0.09857794395215 51	67610.17393615108603 400.21839545031865 600.07940120194502 120.11197369587214 401.03183917829908 001.09059119724857 000.00820782277198 990.05069255626170 690.12664963690540 700.12507146048425 900.26598590474049 900.24263015065217 200.23714968203082 300.21236461456583 100.18686089047348 200.17006217915709 800.17219149393516 300.17657033213745 500.17921172310023 920.16039014130058 500.1897561395463 920.07766347327584 54	67         61         51           0.17393615108603         0.21839545031865         0.15872809711909           0.07940120194502         0.11197369587214         0.05354981829472           0.07940120194502         0.11197369587214         0.05354981829472           1.03183917829908         0.0959119724857         0.05364981829472           0.00820782277198         0.05069255626170         0.02368044821871           0.00820782277198         0.05069255626170         0.02368044821871           0.12664963690540         0.12507146048425         0.12454597279344           0         0.224263015065217         0.29298602326214           0.23714968203082         0.21236461456583         0.29298602326214           0.023714968203082         0.21236461456583         0.25041448471267           0.17821172310023         0.17006217915709         0.20634157716074           0.17921172310023         0.17657033213745         0.17186828346236           30         0.17657033213745         0.178390998635335           0.01897561395463         0.04266323191053         70           0.01897561395463         0.07766347327584         0.11565002863923           0.09857794395215         0.07766347327584         0.11565002863923           0.02925310186986         0.0120244222529

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

			Date of is	sue: 29 September 2020
log(p)	0.05991173983217 06	0.05705782607170 88	0.05892814134969 06	0.05903755733703 27
log(p)	0.00492062089277 53	- 0.02123216305730 09	0.01363006478441 53	- 0.00362781229716 23
log(p)	0.01506213090975 76	- 0.01288673678271 91	0.01826873582759 40	0.00790861155913 90
log(p)	0.01133288594013 20	0.03517878527126 03	- 0.00101016396735 20	0.03236124182408 04
log(p)	0.00517682830585 60	0.02318081655035 85	- 0.01753124094136 09	0.01407383853107 30
log(p)	- 0.13697864042380 60	- 0.12549873297552 30	- 0.14975211056719 50	- 0.13629871172051 70
log(p)	0.06648589379660 10	0.06175670406920 72	0.06915782155993 86	0.06696116027567 27
log(p)	0.08841227789883 30	0.08006864047528 21	0.08984031673510 98	0.08595720824468 91
log(p)	0.06092218513460 66	0.06242698868167 10	0.05558966815467 20	0.05924637681137 36
log(p)			0.07327651895550 10	0.07162322210949 32
log(p)			0.06041012354885 21	0.04900496075393 02
log(p)	0.06610414865301 94	0.07148555595445 57		0
log(p)	0.07865603637227 75	0.06507406043503 56		
log(p)			0.09138613092744 17	0.08744232256586 37
log(p)			- 0.15076237238335 90	- 0.14111001097395 40



log(p)	0.03797303655525 50	0.01399123544029 29		
	0.10463733858903	0.07085220741141	0.10268384217559	0.09113419458164
log(p)	50	83	90	14
	-	-		-
	0.00887962894599	0.05419627241688	0.00163263532894	0.01479110267463
log(p)	99	52	70	79
Total probability				
STRmix	2.975006250	2.730521024	2.600282258	2.841740925
Total				
probability				
Excel	2.975006250	2.730521024	2.600282258	2.841740925

There are no differences in the  $10^{th}$  significant figure. These tests satisfy the acceptance criteria.

#### 4.10. Locus specific amplification efficiency penalties

Acceptance criterion: Expected log(likelihood) is output to 15 sig fig. Precision is checked to 10 sig fig.

The locus amplification variance penalty was calculated in Excel for 20 interpretations from post burn-in extended output. Profiles included single source and mixed, replicates, multikits, drop-in peaks, saturated peaks, assumed contributors, Mx priors.

From version 2.7 onwards,  $\sigma_A^2$  is sampled during the MCMC process from an exponential distribution and a penalty applied. The parameter used to describe an exponential distribution is lambda,  $\lambda$ , and the mean of this distribution is  $1/\lambda$ . Lambda is determined during Model Maker analysis and the mean is used as the Locus Amplification Variance parameter within STRmix<sup>TM</sup> Settings. The penalty can be calculated in Excel using the following formula:

=LOG(EXPON.DIST(LSAE var,  $\lambda$ , 0)) where  $\lambda$  is the LSAE variance parameter sampled within the MCMC process in STRmix<sup>TM</sup>. The penalty is smaller the further the proposed LSAE variance strays from the mean. In the following table five iterations (five chains) of the penalty applied during the MCMC are calculated where 1/ $\lambda$  =0.019:

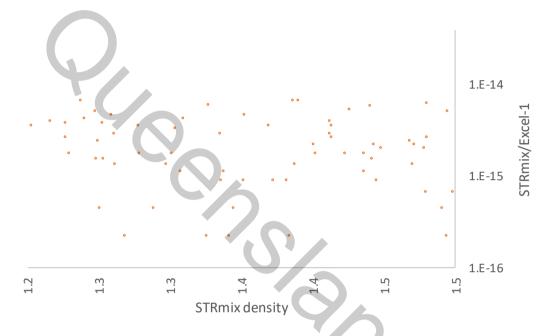
LSAE variance	STRmix penalty	Excel penalty
0.0137523898943288	1.406899712	1.406899712
0.0150988934454997	1.376121867	1.376121867

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 53 of 141

Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

0.0118746972505151	1.449819267	1.449819267
0.0181837156343168	1.305610222	1.305610222
0.0216516826163686	1.226340805	1.226340805

Figure 4.1 shows a zoom of the exponential density unit test.



#### Figure 4.1 Zoom of comparison of exponential density STRmix<sup>™</sup> and MS Excel

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

A penalty is applied to the sampled per locus LSAE parameters. The penalty can be calculated in Excel using the following formula:

=LOG(NORM.DIST(LOG(A<sup>I</sup>),0,SQRT( $\sigma_A^2$ ),0)) where  $\sigma_A^2$  is the sampled LSAE variance per iteration. In the following table one iteration of the per locus penalty are calculated for a 21 locus profile where  $\sigma_A^2$  = 0.0137523898943288 (the first chain in the table above):

Locus	A	Penalty
LSAE_Locus 1 (D3S1358)	1.387069370419550	0.212895381584714
LSAE_Locus 2 (vWA)	1.046227652875870	0.525638950535388
LSAE_Locus 3 (D16S539)	0.862441558791565	0.466498525881671
LSAE_Locus 4 (CSF1PO)	0.888979476867885	0.490477201791106
LSAE_Locus 5 (TPOX)	0.927922190808752	0.515054883256491
LSAE_Locus 7 (D8S1179)	1.280915855888240	0.349180362017557
LSAE_Locus 8 (D21S11)	0.819401670019595	0.413569260661096



Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

		Date of Issue: 29 September 2
LSAE_Locus 9 (D18S51)	0.990669308378352	0.531459255357885
LSAE_Locus 11 (D2S441)	0.980599242876516	0.530577897419838
LSAE_Locus 12 (D19S433)	0.778047132074599	0.344142905082050
LSAE_Locus 13 (TH01)	0.628756882197346	-0.109489466305029
LSAE_Locus 14 (FGA)	0.909069680156644	0.504654226545927
LSAE_Locus 15 (D22S1045)	1.450987997192760	0.119052009602909
LSAE_Locus 16 (D5S818)	1.027934043726750	0.529460398706086
LSAE_Locus 17 (D13S317)	0.878956604944536	0.482146612356248
LSAE_Locus 18 (D7S820)	0.857925004088643	0.461788154110071
LSAE_Locus 19 (SE33)	1.096610658425770	0.506391012674211
LSAE_Locus 20 (D10S1248)	0.862102064281114	0.466151016251318
LSAE_Locus 21 (D1S1656)	1.478547464728710	0.076280196257713
LSAE_Locus 22 (D12S391)	1.112630244413250	0.497798172782929
LSAE_Locus 23 (D2S1338)	1.453755567055920	0.114816220006709
STRmix penalty	8.028543176	
Sum penalties Excel	8.028543176	1
		4

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.11. Penalties

#### 4.11.1. Allele and stutter variance penalties

Acceptance criterion: allele and stutter variance penalties are output to 15 sig fig. Precision is checked to 10 sig fig.

The allele and stutter variance penalties were calculated in Excel for 20 interpretations from post burn-in extended output. Profiles included single source and mixed, replicates, multikits, drop-in peaks, saturated peaks, assumed contributors, Mx priors.

Extended outputs were analysed to check the penalties (or probabilities) for allele and stutter variance. The variances and their penalties are contained within the extended output. A summary of some typical results for allele, back stutter and forward stutter is provided in the following table:

$c^2$	c <sup>2</sup> penalty	$k_{a-1}^2$	$k_{a-1}^2$ penalty
15.1477137911941	-1.121575494	4.44207231525283	-1.327824120
15.1353361900925	-1.121141618	4.41008369263630	-1.328791308



Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

			· · · · · · · · · · · · · · · · · · ·
15.1408686264031	-1.121335281	4.44204914141806	-1.327824813
15.1157058294766	-1.120457948	4.50591579181502	-1.325953028
15.0870371828278	-1.119469307	4.62206500969138	-1.322745162

k <sup>2</sup> <sub>a+1</sub>	$k_{a+1}^2$ penalty	$k_{a-0.2}^2$	$k_{a-0.2}^2$ penalty
7.29505631979703	-1.146842273	1.13840043346852	-0.6587723352
7.33617167573560	-1.148011831	1.14810583375804	-0.6573341392
7.26994719134881	-1.146135775	1.14195938321898	-0.6582406274
7.25507448069025	-1.145720092	1.16922560903803	-0.6543311902
7.31469120565082	-1.147398844	1.15584515134752	-0.6562137631

The allele variance penalty (or probability) is calculated using the following formula:

=LOG(GAMMA.DIST( $c^2$ ,  $\alpha$ ,  $\beta$ , 0))

Where  $\alpha$  and  $\beta$  are in the STRmix<sup>TM</sup> settings for each peak type. The calculated values for the above five iterations is provided in the following tables:

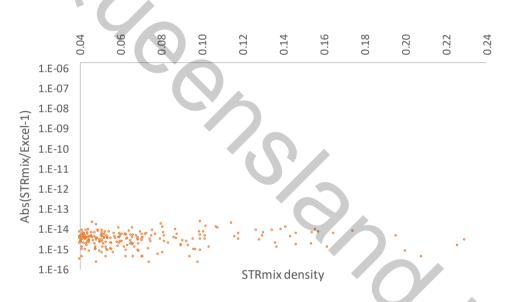
2	2	. 2	12	
c <sup>2</sup> parameters	<sup>c<sup>2</sup></sup> penalty Excel	$k_{a-1}^2$ parameters	$k_{a-1}^2$ penalty Excel	
parametere	portaily Excor	parametere	poneny Exect	
Г(8.45,1.746)	-1.121575494	$\Gamma(1.884, 7.686)$	-1.327824120	
		-		
	-1.121141618		-1.328791308	
	-1.121335281		-1.327824813	
	-1.120457948		-1.325953028	
	-1.119469307	1	-1.322745162	

$k_{a+1}^2$ parameters	$k_{a+1}^2$ penalty Excel	$k_{a=0.2}^2$ parameters	$k_{a-0.2}^2$ penalty Excel
	-1.146842273		-0.6587723352

STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

			•
	-1.148011831	Г(2.189,1.431)	-0.6573341392
	-1.146135775		-0.6582406274
	-1.145720092		-0.6543311902
Γ(2.144, 4.507)	-1.147398844		-0.6562137631

Figure 4.2 below shows a zoom of the gamma density unit test below.



#### Figure 4.2 Zoom of comparison of gamma density STRmix<sup>™</sup> and MS Excel

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.11.2. Drop-in penalty

Acceptance criterion: drop-in penalties are output to 15 sig fig. Precision is checked to 10 sig fig.

The drop-in penalty was calculated in Excel for three interpretations from post burn-in extended output including multiple drop-in peaks, multi kits, and replicates.

The drop-in penalty is replicated in the table below where O=80, Z=75,  $c^2=14.7537$  (mean of  $\Gamma(8.45,1.746)$ ), b =1000,  $f_{di}$  =0.0001. The drop-in penalty has been calculated for both a uniform and gamma drop-in prior (with prior parameters  $\Gamma(5,3)$ ).

E = eVariance $p(O = o   E = e)$	$p(E = e \mid C)$ Gamma prior	$p(E = e \mid C)$ Uniform prior
----------------------------------	----------------------------------	---------------------------------

STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

	$\left(\frac{b}{E_a} + E_a\right)$	NORM.DIST(LOG(O/ <i>E</i> <sub>a</sub> ),0, SQRT( <i>c</i> <sup>2</sup> /var),0)	GAMMA.DIST( <i>E</i> <sub>a</sub> ,5,3,0)	1/(Z-1)
1	1001	1.43837395361617E-53	0.000122862022	0.010101010101
2	502	2.53209549679704E-19	0.001408551767	0.010101010101
3	336.3333333	1.63742241322308E-10	0.005109436683	0.010101010101
97	107.309278350515	1.048861888977	0.00000000138	0.010101010101
98	108.204081632653	1.050051066692	0.00000000103	0.010101010101
99	109.101010101010	1.051044763657	0.00000000077	0.010101010101

The drop-in penalty for a gamma prior is the log of the sum of products of column p(O = o | E = e) and p(E = e | C) for the gamma prior multiplied by  $f_{di}$ . The drop-in penalty for a uniform prior is the log of the sum of products of column p(O = o | E = e) and p(E = e | C) multiplied by  $f_{di}$  for the uniform prior. The following values were obtained from STRmix<sup>TM</sup> and Excel:

Model	STRmix <sup>™</sup> drop-in penalty	Excel
Gamma prior	-4.677289093	-4.677289093
Uniform prior	-4.164180358	-4.164180358

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.11.3. Mx prior penalties

Acceptance criterion: Mx prior penalties are output to 15 sig fig. Precision is expected to 14 sig fig.

The extended output for a three-person mixture was analysed to check the mixture proportion (Mx) priors. The penalty is calculated for each contributor by the following formula:

=LOG(NORM.DIST(Mx,Mx mean,SQRT(Mx var),0))

Where Mx is taken from the extended output and Mx mean and variance from the interpretation set up. The calculated results for five iterations are provided in the following table:

Mx C1	Mx C2	Mx C3

X.S.

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

0.6876289227405140	0.2460821338953720	0.0662889433641133
0.6909319420431470	0.2445278560075910	0.0645402019492624
0.6943718796422450	0.2472549174583470	0.0583732028994076
0.6912375207929320	0.2503470588350160	0.0584154203720522
0.6812439308910470	0.2548074087091730	0.0639486603997798
Mx penalty C1 Excel	Mx penalty C2 Excel	Mx penalty C1 Excel
1.07202946731256	-0.03627948278756	-2.86941392257372
1.08777555433683	-0.11189146070745	-2.97408047062838
1.09901666568106	0.01935137081752	-3.35404553702406
1.08898710575129	0.16016191760807	-3.35138691132720
1.02783642222174	0.34829712367788	-3.00979353991426

Where means of the informed prior = 0.7, 0.3, and 0.2 and the variance = 0.0009765625, 0.00048828125, and 0.0009765625 for contributor 1, 2 and 3, respectively.

Mx penalty STRmix™	Mx penalty sum Excel
-1.833663938	-1.833663938
-1.998196376	-1.998196376
-2.235677500	-2.235677500
-2.102237887	-2.102237887
-1.633659994	-1.633659994

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.12. Replicating Genotype Weights

Acceptance criterion: Precision is expected to at least 10 significant figures.

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The weight for a given genotype set at a locus is calculated as the number of times the genotype set was accepted during the post burn-in MCMC across all chains, divided by the total number of post burn-in iterations. For each post burn-in chain, the genotype set producing the largest log(likelihood) is selected as the starting genotype set. The log(likelihood) for each genotype set is calculated using the mass parameters that were accepted in the final burn-in accept of the corresponding chain. The starting genotype set at each locus is written out to the post burn-in extended output file for each chain (labelled as iteration zero).

A low-level single-source GlobalFiler<sup>™</sup> profile and a two-person unresolvable GlobalFiler<sup>™</sup> mixture were each deconvoluted in STRmix<sup>™</sup> V2.8. A further interpretation of the single-source profile with the GR auto extend function enabled was also performed (500 additional accepts with a GR threshold of 1.2). The profiles were interpreted under the following conditions: 2 chains, 1000 burn-in accepts per chain, 1000 post burn-in accepts per chain (single-source profile) or 500 post burn-in accepts per chain (mixture), extended output enabled. For each interpretation, the weights were replicated across all loci within MS Excel. These were then compared with the STRmix<sup>™</sup> result.

Locus	Contributor 1	Contributor 2	MS Excel	STRmix™
D3S1358	15,16	15,15	0.1769994632	0.1769994632
	16,16	15,15	0.004159957058	0.004159957058
	15,16	15,16	0.04039184111	0.04039184111
	15,15	16,16	0.7784487385	0.7784487385
vWA	18,18	15,16	0.09178743961	0.09178743961
	16,18	15,18	0.09205582393	0.09205582393
	15,18	16,18	0.6155394524	0.6155394524
	15,16	18,18	0.2006172839	0.2006172839

By way of example, the genotype weights at two loci within the mixed profile are reproduced in the table below. Values are presented to 10 significant figures.

The weights calculated in MS Excel match the STRmix<sup>™</sup> result to 10 significant figures and therefore pass the acceptance criterion. While not reproduced here, all other weights calculated in MS Excel also passed the acceptance criterion.



#### 4.13. Likelihood ratios

#### 4.13.1. Posterior mean allele frequencies

Acceptance criterion: Allele frequencies are output to 15 sig fig in the *LR* extended results text file. Precision is checked to 10 sig fig.

The posterior mean frequencies of the alleles including Q were calculated using the following formula:

$$f_{i}' = \frac{x_{i} + \frac{1}{k+1}}{N+1}$$

Where  $x_i$  are the observed allele counts from the database, k the number of possible alleles and N the size of the population database.

The posterior mean of allele Q is calculated using the following formula:

$$f_{Q} = 1 - \sum_{i=1}^{n} f_{i}^{'}$$

Posterior mean frequencies were calculated for each *LR* check described below. An example of a locus is given in the table below for the FBI Caucasian allele frequency database [1].

Locus	Allele	STRmix™	Excel
D3S1358	15	0.288999783	0.288999783
	16	0.305706226	0.305706226
	18	0.055109568	0.055109568
	Q	0.350184421	0.350184421

The results are the same at 10 significant figures, hence these tests pass the acceptance criterion.

#### 4.13.2. *LR* point estimates (sub-sub-source *LR*)

Acceptance criterion: Per locus *LR*s are output to 15 sig fig in the STRmix<sup>™</sup> extended results text file. Precision is checked to 10 sig fig.

The per locus likelihoods  $Pr(E | H_p)$ ,  $Pr(E | H_d)$ , and their ratio (*LR*) are output in the STRmix<sup>TM</sup> interpretation log. The product of these (profile *LR*) are output in the results text files. The individual likelihoods and per locus sub-sub-source *LR* were calculated in Excel for two loci from the weights and posterior mean allele frequencies.

STRmix<sup>™</sup> V2.8 Test Report

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

In the examples below, the population database used for the STRmix<sup>™</sup> analysis was the FBI extended African American subpopulation<sup>2</sup> and  $\theta = 0.03$  within the Balding and Nichols formula,  $\frac{x\theta + (1-\theta)\Pr(A)}{(1+(n-1)\theta)}$ . The first example presented here is for a two person mixture

(C1 and C2) at locus D3S1358 where the POI=15,15. The LR considering the POI is in contributor position 2 is given and there is some dropout (-1 allele) considered.

					( )	
C1		C2		Weight	$\Pr(E \mid H_p)$	$\Pr(E \mid H_d)$
16	18	15	15	0.058262871	0.001691818	0.000174662
16	18	-1	15	0.039974372	0	0.000219835
15	18	15	16	0.003480337	0	2.08669E-05
16	18	15	16	0.303890141	0	0.001606547
18	18	15	16	0.045187905	0	3.0528E-05
-1	18	15	16	0.001760387	0	9.68108E-06
15	18	16	16	0.080841834	0	0.000213689
15	18	-1	16	0.004994618	0	2.74674E-05
15	16	15	18	0.00318752	0	1.91112E-05
16	16	15	18	0.124562866	0	0.000329257
-1	16	15	18	0.002325105	0	1.27867E-05
16	18	15	18	0.162268605	0	0.000219251
15	15	16	18	0.001415978	0	4.24485E-06
-1	15	16	18	0.001119676	0	6.15755E-06
15	16	16	18	0.065844037	0	0.000348091
15	18	16	18	0.034702275	0	4.68883E-05
15	16	18	18	0.060002343	0	4.05363E-05
15	16	-1	18	0.006179131	0	3.39815E-05

The locus likelihoods and LR is given in the table below

	$\Pr(E   H_p)$	$\Pr(E \mid H_d)$	LR
STRmix	0.001691818	0.003363583	0.502980829
Excel	0.001691818	0.003363583	0.502980829

<sup>&</sup>lt;sup>2</sup> Moretti TR, Moreno LI, Smerick JB, Pignone ML, Hizon R, Buckleton JS, et al. Population data on the expanded CODIS core STR loci for eleven populations of significance for forensic DNA analyses in the United States. Forensic Science International: Genetics. 2016;25:175-81.



The results are the same at 10 significant places. Hence these tests pass the acceptance criterion.

The second example is for the same two person mixture (C1 and C2) at locus vWA where the POI=15, 19 and is considered to have dropped out. The *LR* considering the POI is in contributor position 2 is given and there is some drop-in considered.

#	C1		C2		Drop-in	weight
1	14	14	14	14	18	1.20E-04
2	14	18	14	14		0.250961415
3	18	18	14	14		0.020935006
4	-1	18	14	14		0.015950846
5	14	14	-1	14	18	8.51E-05
6	14	18	-1	14		0.040353639
7	18	18	-1	14		0.002247717
8	-1	18	-1	14		0.00119776
9	14	14	14	18		0.339486901
1	-1	14	14	18	3	0.018004049
1	14	18	14	18		0.035300458
1	18	18	14	18		0.002099217
1	-1	18	14	18		0.002095732
1	-1	-1	14	18		0.000957232
1	14	14	18	18		0.144605478
1	-1	14	18	18		0.004290463
1	14	18	18	18		0.004691343
1	14	14	-1	18		0.100031792
1	-1	14	-1	18		0.002014858
2	14	18	-1	18		0.007146821

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

14 18 -1 -1 0.0074243	ſ	2				
			14	-1	-1	0.0074243

щ			1
#	$\Pr(E   H_p)$	$\Pr(E \mid H_d)$	
1	0	1.4957135080E-09	
2	0	3.9165455767E-05	
3	0	2.1489300310E-06	
4	0	1.6530951627E-05	
5	0	1.1364263951E-08	
6	0	8.3642467861E-05	
7	0	4.0315089877E-06	
8	0	2.2473616623E-05	
9	0	5.2980890356E-05	
10	0	3.7317653290E-05	
11	0	1.4494042129E-05	
12	0	8.8186203697E-07	
13	0	7.5178139064E-06	
14	0	8.9802897622E-06	
15	0	1.4843418591E-05	
16	0	7.6953803692E-06	
17	0	1.9707903178E-06	
18	0	1.0366978187E-04	
19	0	3.7804861490E-05	
20	0	2.5637095926E-05	
21	1.1899817106E-04	6.9651205883E-05	LR
Excel	1.1899817106E-04	5.5145087680E-04	0.215791063
STRmix™	1.1899817106E-04	5.5145087680E-04	0.215791063

The results are the same at 10 sig figs. Hence these tests pass the acceptance criterion.



#### 4.13.3. Sub-source LR

Acceptance criterion: Sub-source *LR*s are output to 15 sig fig in the STRmix<sup>™</sup> results text file. Precision is checked to 10 sig fig.

The sub-source *LR* was calculated in Excel for a two person mixture and is given in the table below.

	Contributor order 1		Contributor order 2	
Locus	$\Pr(E \mid H_p)$	$\Pr(E   H_d)$	$\Pr(E \mid H_p)$	$\Pr(E \mid H_d)$
D3S1358	4.1116701786E-05	3.3635827753E-03	1.6918176522E-03	3.3635827753E-03
vWA	1.5342707190E-05	5.5145087680E-04	1.1899817106E-04	5.5145087680E-04
D16S539	7.6236604313E-03	1.4271711960E-02	9.4882877236E-03	1.4271711960E-02
CSF1PO	1.6392640801E-03	1.3843815552E-02	4.2303924846E-03	1.3843815552E-02
TPOX	7.7840377093E-02	7.2785251599E-02	7.7630925479E-02	7.2785251599E-02
D8S1179	2.8835870833E-03	5.7313007386E-03	1.1230327970E-02	5.7313007386E-03
D21S11	2.2395293463E-03	1.7591151013E-04	1.8711779565E-03	1.7591151013E-04
D18S51	2.0005874569E-03	5.6163742982E-03	3.0505975409E-03	5.6163742982E-03
D2S441	1.0751094210E-04	5.8661418394E-04	1.3652411922E-03	5.8661418394E-04
D19S433	9.1990833196E-03	3.3940941804E-03	8.1042663523E-03	3.3940941804E-03
TH01	1.9677404674E-03	2.0352547356E-03	3.4351913935E-03	2.0352547356E-03
FGA	1.4833199495E-02	1.6362797472E-02	1.9119124086E-02	1.6362797472E-02
D22S1045	1.7397073671E-02	4.4192526701E-03	1.2890782596E-02	4.4192526701E-03
D5S818	2.2652392040E-02	1.7359800095E-02	2.7227980432E-02	1.7359800095E-02
D13S317	3.7244779103E-03	2.3356552256E-02	9.1999237730E-03	2.3356552256E-02



Page 65 of 141

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

D7S820	1.1808079152E-01	5.4542882221E-02	9.4906868348E-02	5.4542882221E-02
SE33	3.0749291758E-03	7.4932565371E-03	3.9797124856E-03	7.4932565371E-03
D10S1248	2.9986329596E-02	1.7743060480E-02	3.1880978779E-02	1.7743060480E-02
D1S1656	4.9362431191E-05	2.7258033348E-04	1.6818828677E-04	2.7258033348E-04
D12S391	1.3144820197E-03	1.6038290718E-04	2.1681839774E-03	1.6038290718E-04
D2S1338	2.2759341533E-03	3.7878202855E-03	3.4932323926E-03	3.7878202855E-03
Product	5.4011862364E-55	2.1548744037E-50	1.4474743086E-48	2.1548744037E-50

The sub-source *LR* is replicated below

	$\Pr(E   H_p)$	$\Pr(E   H_d)$	LR		
Excel	7.2373742436E-49	2.1548744037E-50	3.3586060660E+01		
STRmix 3.3586060660E+01					



The results are identical to 10 sig fig. Hence these tests satisfy the acceptance criteria.

#### 4.13.4. LR point estimates with varying theta values

Acceptance criterion: Precision is expected to 10 significant figures.

A full, single-source GlobalFiler<sup>TM</sup> profile was interpreted in STRmix<sup>TM</sup> V2.8. An *LR* was then assigned for the known donor using the FBI extended Caucasian allele frequencies with  $\theta = 0$ ,  $\theta = 0.01$ , and  $\theta = 1.0b(0.8, 110.3)$ . *LRs* were calculated under the same conditions in MS Excel and compared with the STRmix<sup>TM</sup> result. A summary of the locus *LR* results for STRmix<sup>TM</sup> and MS Excel are provided in Table 4.1 below. All results matched those of STRmix<sup>TM</sup> to 10 significant figures and therefore meet the acceptance criteria.

## Table 4.1 Unrelated LRs assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

Locus	STRmix <sup><math>\mathbf{M}</math></sup> : $\theta = 0$	MS Excel: $\theta = 0$
	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0.01$
	STRmix™ <i>θ</i> = 1.0b(0.8, 110.3)	MS Excel: $\theta = 1.0b(0.8, 110.3)$
D3S1358	13.17678247	13.17678247
	12.50011024	12.50011024
	12.68206194	12.68206194
vWA	8881.032754	8881.032754
	235.2591507	235.2591507
	324.0749731	324.0749731
D16S539	92.49465682	92.49465682
	63.03538569	63.03538569
	69.57941153	69.57941153
CSF1PO	5.127380908	5.127380908
	5.056742323	5.056742323
	5.076180837	5.076180837
ТРОХ	3.594355093	3.594355093
	3.567991610	3.567991610
	3.575205421	3.575205421
D8S1179	14.08861398	14.08861398
	13.12725824	13.12725824
	13.38102231	13.38102231
D21S11	11.92934759	11.92934759
	11.37956620	11.37956620
	11.52792910	11.52792910
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STRMIX ©2020 Institute of Environmental Science and Research Limited Page 67 of 141

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020
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		Date of Issue: 29 September 2020
D18S51	81596.19289	81596.19289
	1140.307952	1140.307952
	1661.130752	1661.130752
D2S441	38.47709640	38.47709640
		32.35432331
	32.35432331	33.85951335
<b>D</b> 400400	33.85951335	40.04400040
D19S433	10.01102340	10.01102340
	9.553223002	9.553223002
	9.676109745	9.676109745
TH01	17.42299764	17.42299764
	16.31012062	16.31012062
	16.60747312	16.60747312
FGA	18.56194631	18.56194631
	17.29678327	17.29678327
	17.63375736	17.63375736
D22S1045	48.57263764	48.57263764
	36.95056179	36.95056179
	39.70542178	39.70542178
D5S818	3.494594125	3.494594125
	3.484138310	3.484138310
	3.487043516	3.487043516
D13S317	15.93786153	15.93786153
	14.75448152	14.75448152
	15.06581635	15.06581635
D7S820	15.11246939	15.11246939
	14.24914738	14.24914738
	14.48065258	14.48065258
SE33	408.4053112	408.4053112
	253.1744378	253.1744378
	285.5431956	285.5431956
D10S1248	13.28868260	13.28868260
	11.71568772	11.71568772
	12.12150512	12.12150512
D1S1656	29.80028375	29.80028375
	26.91385029	26.91385029
	27.66920896	27.66920896
	21.00020000	
D12S391	242.0812645	242.0812645



	171.8786379	171.8786379
D2S1338	21.39871316	21.39871316
	19.72850549	19.72850549
	20.17018096	20.17018096
LR total	1.167305498E+34	1.167305498E+34
	3.256835914E+29	3.256835914E+29
	1.279597303E+30	1.279597303E+30

#### 4.13.5. Relatives LRs

Acceptance criterion: Precision is expected to 10 significant figures.

A single-source GlobalFiler<sup>TM</sup> profile was interpreted within STRmix<sup>TM</sup> V2.8. The profile is low level and includes instances of dropout and drop-in within the accepted genotypes. An *LR* was then assigned for one of the known donors. This was done using the FBI extended Caucasian population with  $\theta = 0$ ,  $\theta = 0.01$ , and  $\theta = 1.0b(0.8, 110.3)$ . Relatives *LR*s were replicated in MS Excel across all loci and are summarised in the tables below. All results matched those of STRmix<sup>TM</sup> to 10 significant figures and therefore meet the acceptance criteria.

## Table 4.2: Full sibling *LR*s assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

Locus	STRmix™: <i>θ</i> = 0	MS Excel: $\theta = 0$
	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0$
	STRmix™ <i>θ</i> = 1.0b(0.8, 110.3)	MS Excel: $\theta$ = 1.0b(0.8, 110.3)
D3S1358	2.596913938	2.596913938
	2.571889152	2.571889152
	2.578802879	2.578802879
vWA	2.896550596	2.896550596
	2.860520683	2.860520683
	2.870462329	2.870462329
D16S539	1.940914304	1.940914304
	1.909367902 1.918056275	1.909367902 1.918056275
CSF1PO	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
ТРОХ	2.104454157	2.104454157
	2.064549747	

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020
----------------------------------

		Date of Issue: 29 September 2020
	2.075519013	2.064549747
		2.075519013
D8S1179	2.387221962	2.387221962
	2.368806432	2 200000422
	2.373896015	2.368806432
		2.373896015
D21S11	2.838514707	2.838514707
	2.804988375	2.804988375
	2.814243925	2.814243925
D18S51	Not calculated – no peak information	Not calculated – no peak information
Diocon	at locus	at locus
D2S441	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
D19S433	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
TH01	2.765132872	2.765132872
	2.691362316	2.691362316
	2.711588311	2.711588311
FGA	2.480158558	2.480158558
	2.422472198	2.400100000
	2.438313328	2.422472198
	2.430313320	2.438313328
D22S1045	2.482992765	2.482992765
	2.460643596	
	2.466811549	2.460643596
		2.466811549
D5S818	1.956183218	1.956183218
	1.950804272	1.950804272
	1.952296688	1.952296688
D13S317	2.879051769	2.879051769
Diocon	2.841634892	2.841634892
	2.851940996	2.851940996
D70000		
D7S820	1.554287263	1.554287263
	1.540884576	1.540884576
	1.544582577	1.544582577
SE33	1.893997374	1.893997374
	1.849927030	1.849927030
	1.862006491	1.862006491

Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

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D10S1248	2.983758910	2.983758910
	2.940525819	2.940525819
	2.952414548	2.952414548
D1S1656	3.663951950	3.663951950
	3.598226926	3.598226926
	3.616347578	3.616347578
D12S391	2.564683299	2.564683299
	2.502443326	2.502443326
	2.519527643	2.519527643
D2S1338	3.362307791	3.362307791
C C	3.308211964	3.308211964
	3.323124432	3.323124432
LR total	5.541071255E+06	5.541071255E+06
	4.278883243E+06	4.278883243E+06
	4.596451005E+06	4.596451005E+06

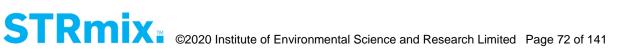
# Table 4.3: Parent/child *LR*s assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

Locus	STRmix <sup>TM</sup> : $\theta = 0$	MS Excel: $\theta = 0$
	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0$
	STRmix <sup>™</sup> θ = 1.0b(0.8, 110.3)	MS Excel: $\theta = 1.0b(0.8, 110.3)$
D3S1358	4.518828452	4 540000450
0351358		4.518828452
	4.408875955	4.408875955
	4.438894133	4.438894133
vWA	6.034768212	6.034768212
	5.802950773	5.802950773
	5.865572672	5.865572672
D16S539	2.278360155	2.278360155
	2.222109536	2.222109536
	2.237462456	2.237462456
CSF1PO	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
ΤΡΟΧ	2.453663358	2.453663358
TPUX		
	2.385009864	2.385009864
	2.403701986	2.403701986
D8S1179	3.712500000	3.712500000
	3.650602410	3.650602410

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Date of Issue: 29	September 2020
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		Date of Issue. 29 September 2020
	3.667597334	3.667597334
D21S11	5.738419619	5.738419619
	5.533598689	5.533598689
	5.589042071	5.589042071
D18S51	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
D2S441	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
D19S433	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
TH01	4.923346113	4.923346113
	4.568425069	4.568425069
	4.661827435	4.661827435
FGA	3.665701873	3.665701873
	3.481905403	3.481905403
	3.531111826	3.531111826
D22S1045	3.946940985	3.946940985
	3.872296114	3.872296114
	3.892757118	3.892757118
D5S818	2.636289666	2.636289666
	2.619785287	2.619785287
	2.624351523	2.624351523
D13S317	5.576592083	5.576592083
	5.385868760	5.385868760
	5.437554273	5.437554273
D7S820	1.589527625	1.589527625
	1.576308078	1.576308078
	1.579960028	1.579960028
SE33	1.882305782	1.882305782
	1.829437612	1.829437612
	1.843830749	1.843830749
D10S1248	5.903790087	5.903790087
	5.684092046	5.684092046
	5.743494141	5.743494141
D1S1656	21.81328546	21.81328546
	18.23592350	18.23592350
	19.10680733	19.10680733



		•
	3.759690169	3.759690169
	3.818862607	3.818862607
D2S1338	11.07056622	11.07056622
	10.15827327	10.15827327
	10.39638295	10.39638295
LR total	5.776549040E+10	5.776549040E+10
	2.717675641E+10	2.717675641E+10
	3.331493416E+10	3.331493416E+10

## Table 4.4: Half sibling, uncle/aunt, niece/nephew, grandparent/grandchild *LR*s assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

Locus	STRmix <sup>TM</sup> : $\theta = 0$	MS Excel: $\theta = 0$
	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0$
	STRmix™ <i>θ</i> = 1.0b(0.8, 110.3)	MS Excel: $\theta$ = 1.0b(0.8, 110.3)
D3S1358	6.269720539	6.269720539
	6.088988110	6.088988110
	6.138246028	6.138246028
vWA	9.291673806	9.291673806
	8.848747106	8.848747106
	8.967909879	8.967909879
D16S539	3.215577675	3.215577675
	3.101201581	3.101201581
	3.132293968	3.132293968
CSF1PO	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
TPOX	4.055342999	4.055342999
	3.859640516	3.859640516
	3.912333668	3.912333668
D8S1179	4.923349203	4.923349203
	4.822917746	4.822917746
	4.850416728	4.850416728
D21S11	8.513402857	8.513402857
	8.152728647	8.152728647
	8.250165983	8.250165983
D18S51	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
L		

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D19S433     N       TH01     8       7       FGA     5	Not calculated – no peak information at locus Not calculated – no peak information at locus 3.214523873 7.481044559 7.673359481 5.882334625	Not calculated – no peak information at locus Not calculated – no peak information at locus 8.214523873 7.481044559 7.673359481
D19S433 N a TH01 8 7 FGA 5	Not calculated – no peak information at locus 3.214523873 7.481044559 7.673359481 5.882334625	Not calculated – no peak information at locus 8.214523873 7.481044559
TH01 8 7 7 FGA 5	at locus 3.214523873 7.481044559 7.673359481 5.882334625	at locus 8.214523873 7.481044559
TH01 8 7 7 FGA 5	3.214523873 7.481044559 7.673359481 5.882334625	8.214523873 7.481044559
7 7 FGA 5	7.481044559 7.673359481 5.882334625	7.481044559
FGA 5	7.673359481 5.882334625	
FGA 5	5.882334625	7.673359481
5		5.882334625
	5.494404834	5.494404834
5	5.597829492	5.597829492
D22S1045 5	5.592865931	5.592865931
5	5.444454619	5.444454619
5	5.484876150	5.484876150
D5S818 3	3.005361933	3.005361933
2	2.990762953	2.990762953
2	2.994808192	2.994808192
D13S317 9	9.522660496	9.522660496
9	9.010254767	9.010254767
9	9.147149473	9.147149473
D7S820 2	2.117726258	2.117726258
2	2.080044803	2.080044803
2	2.090368655	2.090368655
SE33 3	3.444285025	3.444285025
3	3.248697315	3.248697315
3	3.301032215	3.301032215
D10S1248 1	11.68168555	11.68168555
1	10.84767327	10.84767327
1	1.06725698	11.06725698
D1S1656 4	13.59791601	43.59791601
3	35.19775305	35.19775305
3	37.20860795	37.20860795
D12S391 6	6.483064205	6.483064205
6	6.015789631	6.015789631
6	6.139845423	6.139845423
D2S1338 2	20.13501218	20.13501218
1	18.07167294	18.07167294
1	18.60479038	18.60479038
LR total 1	1.194226052E+14	1.194226052E+14
4	4.289408634E+13	4.289408634E+13

5.655106310E+13 5.655106310E+13	 -	
	5 655106310E+13	5 655106310E±13

## Table 4.5: First cousin *LR*s assigned in STRmix<sup>TM</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

Locus	STRmix™: <i>θ</i> = 0	MS Excel: $\theta = 0$
	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0$
Y	STRmix™ θ = 1.0b(0.8, 110.3)	MS Excel: $\theta$ = 1.0b(0.8, 110.3)
D3S1358	7.776233287	7.776233287
	7.522260633	7.522260633
	7.591353567	7.591353567
vWA	12.72562049	12.72562049
	11.99724596	11.99724596
	12.19216519	12.19216519
D16S539	4.048204670	4.048204670
	3.865898496	3.865898496
	3.915199617	3.915199617
CSF1PO	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
TPOX	6.020271549	6.020271549
	5.586764106	5.586764106
	5.701566149	5.701566149
D8S1179	5.882681162	5.882681162
	5.745430780	5.745430780
	5.782929250	5.782929250
D21S11	11.22828923	11.22828923
	10.68029558	10.68029558
	10.82793004	10.82793004
D18S51	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
D2S441	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
D19S433	Not calculated – no peak information	Not calculated – no peak information
	at locus	at locus
TH01	12.33860268	12.33860268



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Date of Issue: 29	September 2020
	•

	10.98178825	10.98178825
	11.33434147	11.33434147
FGA	8.431613098	8.431613098
	7.727648870	7.727648870
	7.913737656	7.913737656
D22S1045	7.066217543	7.066217543
	6.831192171	6.831192171
	6.894858141	6.894858141
D5S818	3.231566509	3.231566509
	3.218653492	3.218653492
	3.222235546	3.222235546
D13S317	14.73654509	14.73654509
	13.57930965	13.57930965
	13.88264017	13.88264017
D7S820	2.539695560	2.539695560
	2.475606392	2.475606392
	2.493062593	2.493062593
SE33	5.886772438	5.886772438
	5.307419065	5.307419065
	5.457658615	5.457658615
D10S1248	22.87556404	22.87556404
	19.87530052	19.87530052
	20.62709734	20.62709734
D1S1656	87.08143793	87.08143793
	65.79835551	65.79835551
	70.69851597	70.69851597
D12S391	9.452087071	9.452087071
	8.594451129	8.594451129
	8.820147503	8.820147503
D2S1338	34.09212053	34.09212053
	29.60167995	29.60167995
	30.74014869	30.74014869
LR total	5.352123921E+16	5.352123921E+16
	1.324779743E+16	1.324779743E+16
	1.925501490E+16	1.925501490E+16

As a further test, a two-person unresolvable mixture was deconvoluted in STRmix<sup>™</sup> V2.8. Peak heights in the evidence input file were reduced to one tenth of their original values to ensure that genotype sets considering allelic dropout and/or drop-in were accepted.



Relatives *LR*s were then assigned at two loci (D16S539 and TPOX) for one of the known donors using the FBI extended Caucasian allele frequencies with  $\theta = 0.01$ . The locus *LR*s, sub-sub-source *LR*s, and sub-source *LR* were all replicated in MS Excel. The results are summarised in the tables below. All results matched those of STRmix<sup>TM</sup> to 10 significant figures and therefore meet the acceptance criteria.

## Table 4.6: Full sibling *LR*s assigned in STRmix<sup>TM</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

LR	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0.01$
Contributor order 1	1.396278882	1.396278882
D16S539 LR		
Contributor order 1	0.7251356073	0.7251356073
TPOX <i>LR</i>		
Contributor order 1	1.012491535	1.012491535
Sub-sub-source LR		
Contributor order 2	1.570563861	1.570563861
D16S539 <i>LR</i>		
Contributor order 2	0.7261279642	0.7261279642
TPOX <i>LR</i>		$\sim$
Contributor order 2	1.140430339	1.140430339
Sub-sub-source LR		· · · O
Sub-source <i>LR</i>	1.080390646	1.080390646

## Table 4.7: Parent/child *LR*s assigned in STRmix<sup>M</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

LR	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0.01$
Contributor order 1	1.528189960	1.528189960
D16S539 <i>LR</i>		



		Date of Issue. 29 September 2020
Contributor order 1	0.6770140500	0.6770140500
TPOX <i>LR</i>		
Contributor order 1	1.034606074	1.034606074
Sub-sub-source <i>LR</i>		
Contributor order 2	1.802152200	1.802152200
D16S539 <i>LR</i>		
Contributor order 2	0.6782779461	0.6782779461
TPOX LR		
Contributor order 2	1.222360093	1.222360093
Sub-sub-source LR	6	
Sub-source <i>LR</i>	1.132013345	1.132013345

## Table 4.8: Half sibling, uncle/aunt, niece/nephew, grandparent/grandchild *LR*s assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

LR	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0.01$
Contributor order 1	1.652362299	1.652362299
D16S539 <i>LR</i>		
Contributor order 1	0.6580819044	0.6580819044
TPOX <i>LR</i>		
Contributor order 1	1.087389729	1.087389729
Sub-sub-source <i>LR</i>		
Contributor order 2	2.016202374	2.016202374
D16S539 <i>LR</i>		
Contributor order 2	0.6591199490	0.6591199490
TPOX <i>LR</i>		
Contributor order 2	1.328919206	1.328919206
Sub-sub-source <i>LR</i>		



STRmix™ V2.8 Test Report Issued by Institute of Environmental Science and Research Limited of le 20 0 2020 Da

ate of Issue: 29 September 202	2
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Sub-source LR	1.210655309	1.210655309

#### Table 4.9: First cousin *LR*s assigned in STRmix<sup>™</sup> V2.8 and using MS Excel. Values are displayed to 10 significant figures.

LR	STRmix™: <i>θ</i> = 0.01	MS Excel: $\theta = 0.01$
Contributor order 1	1.722336094	1.722336094
D16S539 <i>LR</i>		
Contributor order 1	0.6490074229	0.6490074229
TPOX LR		
Contributor order 1	1.117808910	1.117808910
Sub-sub-source LR	0	
Contributor order 2	2.143499160	2.143499160
D16S539 <i>LR</i>		
Contributor order 2	0.6499411377	0.6499411377
TPOX <i>LR</i>		
Contributor order 2	1.393148283	1.393148283
Sub-sub-source <i>LR</i>		
Sub-source LR	1.256980023	1.256980023

#### 4.13.6. Unified *LR*

Acceptance criterion: Precision is expected to 10 significant figures.

Formula for the prior proportions used when assigning a unified *LR* are given in the following table for a population of size N=1,000,000 with average number of children per family n=4.

Table 4.10: Relationship prior proportions calculated in STRmix <sup>™</sup> and MS Excel. Values are	
displayed to 10 significant figures.	

Relationship	Prior	STRmix™ priors	MS Excel priors
Siblings	<i>n</i> -1	3.00000000E-6	3.000000000000E-06
Children	2n/3	2.666666667E-6	2.666666667E-06
Parents	4/3	1.333333333E-6	1.333333333E-06



Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

		-	
Uncle/Aunt	4(n-1)/3	4.00000000E-6	4.00000000E-06
Niece/Nephew	2 <i>n</i> ( <i>n</i> -1)/3	8.00000000E-6	8.00000000E-06
Grandparent	4/3	1.333333333E-6	1.333333333E-06
Grandchild	<i>n</i> ²/3	5.333333333E-6	5.333333333E-06
Cousins	2n(n-1)	2.40000000E-5	2.40000000E-05
Unrelated	1-∑(relatives	0.9999503333	0.9999503333
	proportions)		

The MS Excel priors matched those of STRmix<sup>TM</sup> to 10 significant figures. The results therefore pass the acceptance criterion.

Unified point estimate *LR*s were assigned for the single-source and mixed DNA profiles examined within Section 4.13.5. *LR*s were assigned using the FBI extended Caucasian allele frequencies with  $\theta = 0.01$ . For the single-source profile, unified *LR*s were also assigned with  $\theta = 0$  and  $\theta = 1.0b(0.8,110.3)$ . Unified *LR*s were manually assigned in MS Excel and compared with the STRmix<sup>TM</sup> result. All results matched those of STRmix<sup>TM</sup> to 10 significant figures and therefore pass the acceptance criterion.

## Table 4.11: Point estimate unified *LR*s assigned for the single-source and mixed profiles examined in Section 4.13.5. Values are displayed to 10 significant figures.

	STRmix™ unified <i>LR</i>	MS Excel unified <i>LR</i>
Single-source profile, $\theta=0$	1.846787017E+12	1.846787017E+12
Single-source profile, $\theta = 0.01$	1.425994166E+12	1.425994166E+12
Single-source profile, $\theta = 1.0b(0.8)$ ,	1.531867755E+12	1.531867755E+12
110.3)		
Mixed profile	1.308981876	1.308981876
(D16S539 and TPOX only) $\theta = 0.01$		0.0

#### 4.13.7. Stratified *LR*

Acceptance criterion: Stratified *LR* are output to 15 sig fig in the STRmix<sup>TM</sup> extended results text file. Precision is checked to 10 sig fig.

A stratified *LR* is calculated from the individual sub population *LR*s using the following formula:



$$= \frac{\sum_{pop} \Pr(O \mid pop, H_p) P_{pop}}{\sum_{pop} \Pr(O \mid pop, H_d) P_{pop}}$$

A stratified sub-source *LR* for the two person mixed DNA profile described above is summarised below (unrelated, related and unified propositions) where the African American and Caucasian allele frequencies were used in the proportions 0.3 and 0.7, respectively.

Relationship	STRmix	Excel
Unrelated	1.9407763285E+01	1.9407763285E+01
Siblings	1.5923454030E-01	1.5923454030E-01
Parent Child	1.1307943494E-01	1.1307943494E-01
Half sibs, etc	5.8994241320E-01	5.8994241320E-01
Cousins	2.2995226541E+00	2.2995226541E+00
Unified	1.9407686155E+01	1.9407686155E+01

These results are identical and hence these tests satisfy the acceptance criteria.

#### 4.14. Highest posterior density

#### 4.14.1. HPD for unrelated and related *LR*s

Acceptance criterion: HPD *LR* are output to 15 sig fig in the STRmix<sup>TM</sup> results text file. Precision is checked to 10 sig fig.

The sub-source 99% 1-sided lower HPD array is available in the extended output text files for unrelated, related and unified propositions. The HPD *LR* for the two person mixed DNA profile described above is summarised below (unrelated, related and unified propositions).

Relationship	STRmix	Excel
Unrelated	6.909201843E+18	6.909201843E+18
Siblings	6.281831087E+06	6.281831087E+06
Parent Child	7.487164996E+09	7.487164996E+09
Half sibs, etc	1.241465534E+13	1.241465534E+13
Cousins	2.339834912E+15	2.339834912E+15



Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

Unified	3.136910754E+10	3.136910754E+10

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.14.2. Stratified HPD for unrelated and related LRs

Acceptance criterion: HPD *LR* are output to 15 sig fig in the STRmix<sup>TM</sup> results text file. Precision is checked to 10 sig fig.

The stratified sub-source 99% 1-sided lower HPD array is available in the extended output text files for unrelated, related and unified propositions. The HPD *LR* for the two person mixed DNA profile described above is summarised below (unrelated, related and unified propositions) where the African American and Caucasian allele frequencies were used in the proportions 0.7 and 0.3, respectively.

Relationship	STRmix	Excel
Unrelated	3.812452983E+19	3.812452983E+19
Siblings	1.329238672E+07	1.329238672E+07
Parent Child	2.493543181E+10	2.493543181E+10
Half sibs, etc	4.806028524E+13	4.806028524E+13
Cousins	1.021602848E+16	1.021602848E+16
Unified	3.946603851E+13	3.946603851E+13

There are no differences in the 10<sup>th</sup> significant figure. These tests satisfy the acceptance criteria.

#### 4.14.2.1. HPD unit test – allele frequency resampling

Acceptance criterion: The STRmix 99 percentiles are within the range of the simulated 99 percentiles from Excel.

A single source, single locus (D16S539), profile was run that produced split weights for a previously unobserved allele and Q, and for the 12 allele and Q. Theta was set at 0.01. The minimum resampled count (MRC) was either turned off or set to 1. 100 sets of the 99% lower bound on 10,000 replications of the allele frequency resampling process (gamma.inv) were run in Excel using the 'whatif' function. The STRmix 99% lower bound was compared with these. The results were:

Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

	Unobserved	Unobserved	12 allele	12 allele
	MRC Off	MRC = 1	MRC = Off	MRC = 1
STRmix 99%	115.872684	115.872684	3.9869096	3.985682
Excel 99% summary				
Min	113.49947	113.200714	3.951802	3.958924
0.05%ile	116.843362	114.2515769	3.966958	3.977519
Average	115.585158	115.7030782	3.986512	4.002212
0.95%ile	114.2587297	117.0724748	4.004792	4.020871
Max	117.6452517	117.2563573	4.01282	4.029905

These tests satisfy the acceptance criteria.

#### 4.14.2.2. HPD unit test - weight resampling

Acceptance criterion: The STRmix 99 percentiles are within the range of the simulated 99 percentiles from Excel.

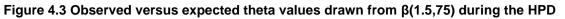
A single source, single locus (D16S539), profile was run that produced split weights for a previously unobserved allele and Q, and for the 12 allele and Q. Theta was set at 0.01. The weight sampling was enabled and allele frequency resampling disabled. 100 sets of the 99% lower bound on 10,000 replications of the allele frequency resampling process (gamma.inv) were run in Excel using the 'whatif' function. The STRmix 99% lower bound was compared with these. The results were:

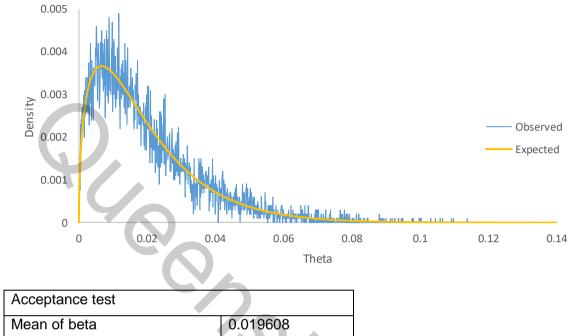
STRmix 99%	4.485382
Excel Min	4.472459
Excel 0.05%ile	4.475923
Excel Average	4.48632
Excel 0.95%ile	4.494006
Excel Max	4.499031

These tests satisfy the acceptance criteria.

### 4.14.3. Sampling theta values during HPD

The extended output for a *LR* calculation for a single source profile was enabled. The prior used for the theta beta distribution within the HPD was 1.0b(1.5,75). Theta values for ten thousand HPD iterations were recovered from the extended output and imported into Excel. The theta values sampled from the posterior distribution during the HPD are plotted in Figure 4.3 with the expected values sampled from a beta distribution.





Mean of beta	0.019608	
Average of sample	0.019625	
Count of sample	10000	
sd of binomial estimate	0.001386	
Accept/reject		
$diff \le 1.96 \times \sqrt{\frac{p(1-p)}{N}}$	yes	0

These tests satisfy the acceptance criteria.

#### 4.15. Gaussian walk

In a Gaussian walk the size of the step for any given variable is sampled randomly from  $\sim N(\text{current parameter state}, sd^2)$ . The *sd* values for the mass and variance parameters optimised during a STRmix<sup>TM</sup> interpretation are given in column 2 of

Table 4.12. Within this table, RWSD refers to the random walk standard deviation parameter, which is located within the STRmix<sup>™</sup> run settings (and also within the default settings). A RWSD value of 0.005 is used by default; this has previously been found to provide the best performance in terms of precision and run-time.

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Variable	Standard deviation (sd)
t <sub>n</sub> (template)	Maximum of:
	<i>RWSD</i> x (highest peak - <i>Z</i> )
	or
	<i>RWSD</i> x 10 x <i>Z</i>
	Where $Z$ is the detection threshold
d <sub>n</sub> (degradation)	RWSD x degradation max
A <sup>I</sup> (locus amplification	2 × RWSD
efficiency)	
Allele and stutter variance	$RWSD \times 1.96 \sqrt{\alpha \beta^2}$
constants ( $c^2$ and $k^2$ )	
LSAE variance	$100 \times RWSD \times \lambda$
Ry and Bk (replicate and kit	5 × RWSD
efficiencies)	

#### Table 4.12 Summary of step size in the Gaussian walk

Single-source and mixed DNA profiles were interpreted within STRmix<sup>™</sup> V2.8. The default run settings were changed to: 2 chains, 1000 burn-in accepts per chain, 1000 post burn-in accepts per chain, and extended output was enabled. The option to include rejected iterations within the extended output files was also enabled. Following interpretation, the distribution of step sizes taken by STRmix<sup>™</sup> for each parameter was examined and compared with expectation. The test set included interpretations with multiple kits, replicates, and conditioned profiles. One interpretation was carried out using a reduced RWSD of 0.0005 to confirm that the step sizes adjusted accordingly. Stepping during burnin was also examined.

By way of example, plots of observed versus expected step size for all mass and variance parameters following interpretation of a two-person GlobalFiler<sup>™</sup> mixture are provided in Figure 4.4. As can be seen in the plots below, the distributions of observed step size correspond with expectation.

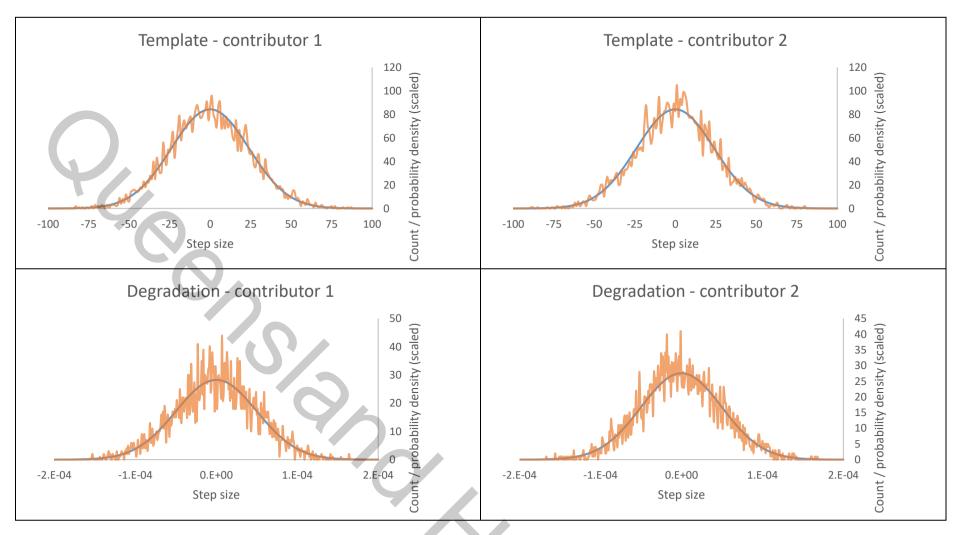
Within the extended output, the parameter values for replicate efficiency ( $R_y$ ) and kit efficiency ( $B_k$ ) are post-processed so that the sum of their logs is zero. The pre-processed values are not output within the extended output results. As such, stepping for these parameters was unable to be verified.

Acceptance criterion: Average step size  $\leq 2.58 \times \frac{sd}{\sqrt{N}}$ 

All profiles and parameters examined satisfied the above acceptance criterion.

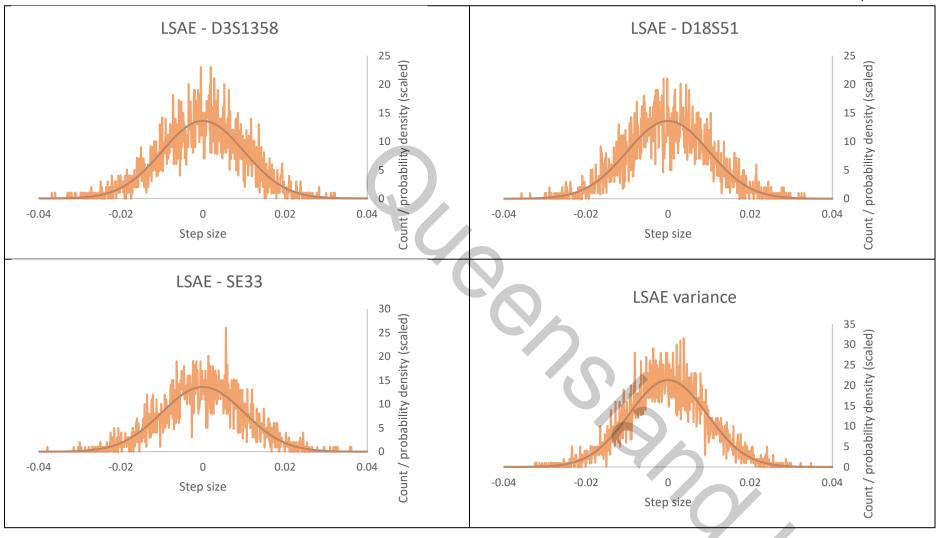
STRMIX ©2020 Institute of Environmental Science and Research Limited Page 87 of 141

Figure 4.4 Observed (orange) versus expected (blue) step size distributions following interpretation of a two-person GlobalFiler™ mixture within STRmix™.



**STRmix**:

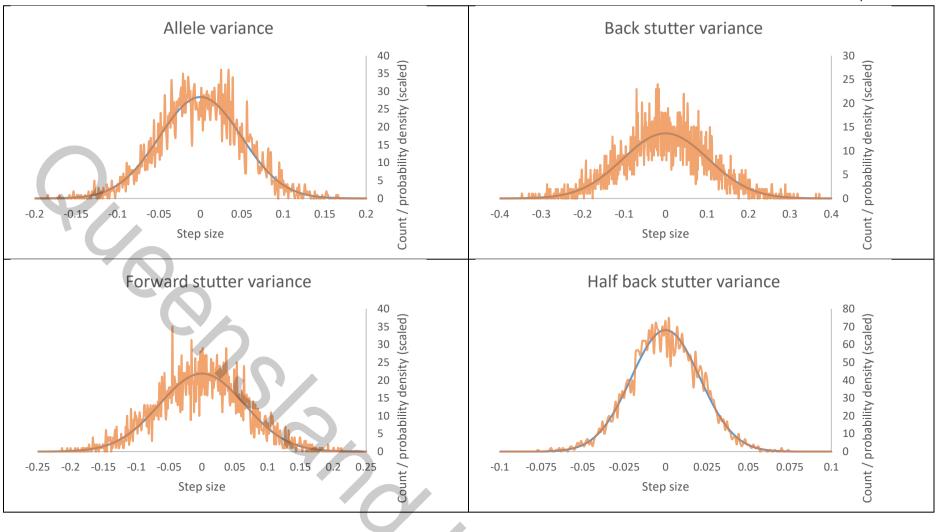
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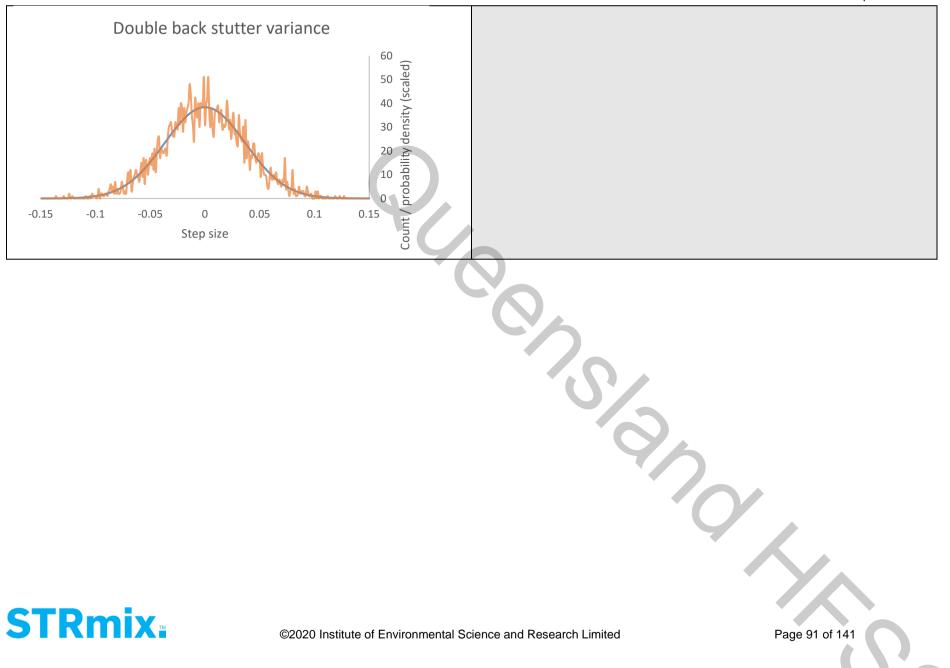


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**STRmix** 

Page 89 of 141





	Template – Contributo r 1	Template – Contribu tor 2	Degradation – Contributo r 1	Degradation – Contribut or 2	LSAE – D3S1358	LSAE – D18S51	LSAE – SE 33
Average step size	-0.114505321	0.237879385	-2.58256E-07	-3.25231E-07	-9.23371E-05	0.000112096	0.00019051 7
Count	5074	5074	3543	3456	3406	3406	3406
Standard deviati on	24.005	24.005	0.00005	0.00005	0.01	0.01	0.01
Accept/reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	LSAE variance	Allele variance	Back stutter variance	Forward stutter variance	Half back stutter variance	Double back stutter variance	
Average step size	-7.85968E-05	0.000809201	-0.002683601	-0.00067457	-0.000287045	-0.00027886	
Count	5074	3543	3543	3543	3543	3543	
Standard deviati on	0.0095	0.049739186	0.103387231	0.064673449	0.02074859	0.036875127	
Accept/reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	



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#### 4.16. **Post burn-in diagnostics**

There are a number of post burn-in diagnostics included in the STRmix<sup>™</sup> report. These may be used to assess the reliability of an interpretation and are discussed further in the STRmix<sup>™</sup> User's Manual. A number of profiles were interpreted in STRmix<sup>™</sup> V2.8 and all post burn-in diagnostics replicated 'by hand' in MS Excel. The profiles examined include single-source profiles, mixtures, mixtures interpreted using a conditioning profile, replicates, multi-kit interpretations (with/without degradation fixed), saturated profiles (single-source and mixtures), and profiles interpreted with the GR auto-extend function enabled. Representative results for each of the diagnostics are reproduced below. Values have been provided to 10 significant figures.

#### 4.16.1. Gelman-Rubin convergence diagnostic

Acceptance criterion: Precision is expected to at least 10 significant figures.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The GR convergence diagnostic was calculated using the method described in the STRmix<sup>™</sup> User's Manual.

	MS Excel	STRmix™
GR	1.552123832	1.552123832

The values calculated match to 10 significant figures and pass the acceptance criterion.

The same profile was reinterpreted with the GR auto continue function enabled (50 additional post burn-in accepts per chain with GR threshold = 1.2). The GR was replicated in MS Excel and compared with the STRmix<sup>TM</sup> result. The initial GR (prior to extension) as well as the final GR post-extension were both replicated.

	MS Excel	STRmix™	5
GR (initial)	1.552123832	1.552123832	
GR (post-extension)	1.021655494	1.021655494	

The values calculated match to 10 significant figures and pass the acceptance criterion.

GR diagnostics calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion.



#### 4.16.2. Effective sample size

Acceptance criterion: Precision is expected to at least 10 significant figures.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>TM</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. ESS was calculated in MS Excel using the method described in the *STRmix<sup>TM</sup> V2.8 User's Manual.* The results were then compared with the STRmix<sup>TM</sup> output and are summarised in the table below.

	MS Excel	STRmix™
Total ESS	44.75003171	44.75003171

The values calculated match to 10 significant figures and pass the acceptance criterion.

Total ESS calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion.

#### 4.16.3. Effective sample size thinning

Acceptance criterion: ESS thinning values are given to 2 decimal places and are therefore expected to be precise to 2 decimal places. Precision for ESS array files and Total ESS is expected to at least 10 significant figures.

A single-source multi-kit sample (GlobalFiler<sup>™</sup> and Identifiler<sup>™</sup> Plus) was interpreted in STRmix<sup>™</sup> V2.8 with ESS thinning amount set to 500 for testing purposes (note: by default ESS thinning amount in the STRmix<sup>™</sup> software is set to 100,000 and is not customisable).

Both input files were analysed at 50 rfu with the D12S391 locus ignored due to an unresolved back stutter peak in the GlobalFiler<sup>™</sup> profile. The default run settings were changed to: 2 chains, 1000 burn-in accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled.

When ESS thinning amount was set to 500, MCMC probability values were thinned to every 3.6<sup>th</sup> iteration within chain 1 and to every 2.22<sup>nd</sup> iteration within chain 2. These thinning values were replicated within MS Excel and compared to the STRmix output. The results are summarised in the table below.

	MS Excel	STRmix™
ESS thinning Chain1	3.5960	3.60
ESS thinning Chain2	2.2160	2.22

The thinned ESS value calculated for each chain matches that used by STRmix when rounded to 2 decimal places and passes the acceptance criterion.

The contents of the STRmix ESS array files after thinning were were replicated within MS Excel. While not reproduced here, all values calculated matched the STRmix<sup>™</sup> result to 10 significant figures and therefore pass the acceptance criterion.

Total ESS was calculated in MS Excel using the method described in the *STRmix*<sup>TM</sup> V2.8 User's Manual. The results were then compared with the STRmix<sup>TM</sup> output and are summarised in the table below.

Y O	MS Excel	STRmix™	
Total ESS	12.12428779	12.12428779	

The total ESS value calculated matches to 10 significant figures and passes the acceptance criterion.

#### 4.16.4. Total iterations

Acceptance criterion: Total iterations is output as an integer in STRmix<sup>™</sup>. Precision is expected to zero decimal places.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The total number of iterations performed is equal to the sum of the post burn-in iterations across all chains. This was calculated in MS Excel and compared with the STRmix<sup>™</sup> result as summarised in the table below.

	MS Excel	STRmix™
Chain 1	741	741
Chain 2	1192	1192
Total iterations	1933	1933

The values calculated match to 0 decimal places and pass the acceptance criterion.

Total iterations calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to 0 decimal places and therefore pass the acceptance criterion.



#### 4.16.5. Acceptance rate

Acceptance criterion: Acceptance rate is output to 2 decimal places in STRmix<sup>™</sup>. Precision is expected to 2 decimal places.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>TM</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The acceptance rate is calculated as the total number of post burn-in accepts divided by the total iterations. The reciprocal is then taken and is presented within the STRmix<sup>TM</sup> report as an acceptance rate of 1 in *x*. The acceptance rate was calculated in MS Excel and compared with the STRmix<sup>TM</sup> result as summarised in the table below.

	MS Excel	STRmix™
Acceptance rate	1 in 19.33	1 in 19.33

The values calculated match to 2 decimal places and pass the acceptance criterion.

The acceptance rate calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to 2 decimal places and therefore pass the acceptance criterion.

#### 4.16.6. Average log(likelihood)

Acceptance criterion: Precision is expected to a minimum of 10 significant figures.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the average log(likelihood) in MS Excel.

		×	
	MS Excel	STRmix™	
Average log(likelihood)	56.08947047	56.08947047	

The values calculated match to 10 significant figures and pass the acceptance criterion.

Average log(likelihood) values calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion.

#### 4.16.7. Template

Acceptance criterion: Template is output as an integer in STRmix<sup>™</sup>. Precision is expected to 0 decimal places.

In STRmix<sup>™</sup> V2.8, per contributor template is calculated by first finding the average per contributor template within each chain, then taking the average of the per chain averages.

This is the value reported within the PDF report. Prior to STRmix<sup>™</sup> V2.8 per contributor template was instead calculated by finding the mode within each chain, then taking the average of the per chain modes. STRmix<sup>™</sup> V2.8 still performs these calculations and outputs the results in the Results.txt file. Prior to calculating the per chain modes, accepted template values are allocated into bins of width equal to the saturation threshold divided by 1000. These bins span from 0 rfu to the saturation threshold. If a multi kit interpretation is performed where the kits have different saturation threshold values, the maximum saturation threshold is used.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate per contributor template using both approaches. The results were then compared with the STRmix<sup>™</sup> output as summarised in the tables below.

		MS Excel	STRmix™
Contributor 1	Chain 1 average	1057.280862	
	Chain 2 average	1107.735453	
	Average of averages	1083	1083
Contributor 2	Chain 1 average	14153.56092	
	Chain 2 average	14004.18052	
	Average of averages	14079	14079
Contributor 3	Chain 1 average	1320.491768	X
	Chain 2 average	1147.189429	
	Average of averages	1234	1234

		MS Excel	STRmix™
Contributor 1	Chain 1 mode	1080	
	Chain 2 mode	1050	
	Average of modes	1065	1065
Contributor 2	Chain 1 mode	14250	
	Chain 2 mode	14100	



			•
	Average of modes	14175	14175
Contributor 3	Chain 1 mode	1260	
	Chain 2 mode	1170	
	Average of modes	1215	1215

The values calculated match to 0 decimal places and pass the acceptance criterion.

Per contributor template values were calculated using both approaches for all other profiles in the test set. All calculated values matched the corresponding STRmix<sup>™</sup> result to 0 decimal places and therefore pass the acceptance criteria.

As a further test, several saturated profiles were examined. Posterior template values calculated as the average of the per chain averages will remain unchanged for saturated contributors. However, where the mode of a contributor's template falls within the last bin, STRmix<sup>™</sup> will instead use the average template amount for that contributor calculated within the affected chain. The results for one of the saturated profiles examined are provided below. The saturation threshold was reduced to 3000 rfu so that contributor 1 would be 'saturated' whilst contributors 2 and 3 would be on-scale.

		MS Excel	STRmix™
Contributor 1	Chain 1 average	10241.45692	
	Chain 2 average	10958.78434	
	Average of averages	10600	10600
Contributor 2	Chain 1 average	2147.191815	
	Chain 2 average	1382.160294	
	Average of averages	1765	1765
Contributor 3	Chain 1 average	106.8476310	
	Chain 2 average	127.8852240	
	Average of averages	117	117

		MS Excel	STRmix™
Contributor 1	Chain 1 mode	3000*	

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		(10241.45692)	
	Chain 2 mode	3000*	
		(10958.78434)	
	Average of modes	10600	10600
Contributor 2	Chain 1 mode	2208	
	Chain 2 mode	1410	
	Average of modes	1809	1809
Contributor 3	Chain 1 mode	102	
	Chain 2 mode	129	
	Average of modes	116	116

The values calculated match to 0 decimal places and pass the acceptance criterion. Note that both of the per chain modes for contributor 1 fall within the last bin (in this case, 3000 rfu). The relevant values have been indicated in the table above using an asterisk. STRmix<sup>™</sup> will replace these values with the per chain averages. These values have been reproduced here in parentheses. The average of these values is 10,600 rfu and matches the STRmix<sup>™</sup> result. These results demonstrate that STRmix<sup>™</sup> is performing as expected.

#### 4.16.8. Mixture proportions

Acceptance criterion: Precision is expected to a minimum of 10 significant figures.

Mixture proportions are calculated using the per contributor template values described above. The mixture proportions included in the STRmix<sup>™</sup> PDF report use the per contributor template values calculated as the average across the per chain averages. An additional set of mixture proportions are included in the Results.txt file; these are calculated using template values found as the average across the per chain modes. For each approach, per contributor template values are summed within each chain. Per chain mixture proportions are then calculated for each contributor as a proportion of their template out of the total amount. For each contributor, the per chain mixture proportions are then averaged across all chains to produce the reported values.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the mixture proportions using both approaches. The results were then compared with the STRmix<sup>™</sup> output as summarised in the tables below.

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 99 of 141

Calculated using template - average of per chain averages:

	MS Excel	STRmix™
Contributor 1 <i>Mx</i>	6.604316751	6.604316751
Contributor 2 <i>Mx</i>	85.87394061	85.87394061
Contributor 3 Mx	7.521742636	7.521742636

Calculated using template – average of per chain modes:

	MS Excel	STRmix™
Contributor 1 Mx	6.471884640	6.471884640
Contributor 2 Mx	86.14608818	86.14608818
Contributor 3 <i>Mx</i>	7.382027178	7.382027178

The values calculated match to at least 10 significant figures and pass the acceptance criterion.

Mixture proprotions calculated for all other mixtures in the test set match the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion.

#### 4.16.9. Degradation

Acceptance criterion: Precision is expected to a minimum of 10 significant figures.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate per contributor degradation (exponential parameter as well as the linear approximation). The results were then compared with the STRmix<sup>™</sup> output as summarised in the table below.

		MS Excel	STRmix™
Contributor 1	Exponential parameter	0.001653107154	0.001653107154
	Linear approximation	1.519887062	1.519887062
Contributor 2	Exponential parameter	0.003172007947	0.003172007947

	Linear approximation	33.06194547	33.06194547
Contributor 2	Exponential parameter	8.979763817E-04	8.979763817E-4
	Linear approximation	1.025475000	1.025475000

The values calculated match to a minimum of 10 significant figures and pass the acceptance criterion.

Per contributor degradation parameters calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion. Of note, a multi-kit interpretation where degradation was fixed between kits produced identical degradation parameters for each kit, as expected.

### 4.16.10. Allele, stutter, and LSAE variance constants

Acceptance criterion: Precision is expected to a minimum of 10 significant figures.

The allele, stutter, and LSAE variance diagnostics output in the STRmix<sup>™</sup> report are each calculated as the average across all accepted values within the post burn-in MCMC. These parameters are kit specific and are described further in the STRmix<sup>™</sup> User's Manual.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the average posterior variance parameters. The results were then compared with the STRmix<sup>™</sup> output as summarised in the table below.

	MS Excel	STRmix™
Allele variance	24.54215850	24.54215850
Back stutter variance	16.19567554	16.19567554
Forward stutter variance	17.45763414	17.45763414
LSAE variance	0.01702957894	0.01702957894

The values calculated match to a minimum of 10 significant figures and pass the acceptance criterion.



Posterior variance parameters calculated for all other profiles in the test set matched the STRmix<sup>™</sup> result to a minimum of 10 significant figures and therefore pass the acceptance criterion.

#### 4.16.11. Locus Specific Amplification Efficiencies

Acceptance criterion: LSAE is output as a percentage to 0 decimal places in STRmix<sup>™</sup>. Precision is expected to 0 decimal places.

The posterior locus specific amplification efficiencies output in the STRmix<sup>™</sup> report are each calculated as are the average across all accepted values in the post burn-in MCMC. These parameters are kit and locus specific as described in User's Manual.

A three-person PowerPlex® Fusion mixture was interpreted in STRmix<sup>™</sup> V2.8 conditioning on one of the known donors. The default run settings were changed to: 2 chains, 1000 burnin accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the average posterior LSAE at three loci. The results were then compared with the STRmix<sup>™</sup> output as summarised in the table below.

	MS Excel	STRmix™
Penta D LSAE	78%	78%
D7S820 LSAE	119%	119%
D22S1045 LSAE	88%	88%

The values calculated match to 0 decimal places and pass the acceptance criterion.

The posterior LSAE at three loci was replicated manually for all other profiles in the test set. All posterior LSAE values calculated matched the STRmix<sup>TM</sup> result to 0 decimal places and therefore pass the acceptance criterion.

#### 4.16.12. Replicate efficiency

Acceptance criterion: Replicate efficiency is output as a percentage to 0 decimal places in STRmix<sup>™</sup>. Precision is expected to 0 decimal places.

The replicate efficiency value is the average of the accepted post burn-in values and scale all peaks up or down between replicates (where appropriate) as described in the STRmix<sup>™</sup> User's Manual.

A single-source GlobalFiler<sup>™</sup> profile with replicate evidence inputs was interpreted in STRmix<sup>™</sup> V2.8. The default run settings were changed to: 2 chains, 1000 burn-in accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the posterior replicate efficiency for both PCR replicates. The results were then compared with the STRmix<sup>™</sup> output as summarised in the table below.

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 102 of 141

	MS Excel	STRmix™
Replicate 1 efficiency	65%	65%
Replicate 2 efficiency	154%	154%

The values calculated match to 0 decimal places and pass the acceptance criterion.

#### 4.16.13. **Kit efficiency**

Acceptance criterion: Kit efficiency is output as a percentage to 0 decimal places in STRmix<sup>™</sup>. Precision is expected to 0 decimal places.

The kit efficiency values are the average of the accepted post burn-in values and scale all peaks up or down between kits (where appropriate) as described in the STRmix<sup>™</sup> User's Manual.

A multi kit interpretation of a single-source profile tested using the Identifiler<sup>™</sup> Plus and GlobalFiler<sup>™</sup> multiplex kits was undertaken in STRmix<sup>™</sup> V2.8. The default run settings were changed to: 2 chains, 1000 burn-in accepts per chain, 50 post burn-in accepts per chain, and extended output was enabled. The extended output files were used to calculate the posterior replicate efficiency for both PCR replicates. The results were then compared with the STRmix<sup>™</sup> output as summarised in the table below.

	MS Excel	STRmix™
Identifiler™ Plus kit efficiency	54%	54%
GlobalFiler™ kit efficiency	186%	186%

The values calculated match to 0 decimal places and pass the acceptance criterion.

#### 4.17. Mix to Mix function

Acceptance criterion: Precision is expected to at least 10 significant figures.

#### Test 1

As an initial test of the Mix to Mix function, an unambiguous single-source GlobalFiler™ profile was compared with a two-person GlobalFiler<sup>™</sup> major/minor mixture. The donor to the single-source profile is the known minor donor to the mixed profile. The STRmix<sup>™</sup> Mix to Mix function was used to compare prior deconvolutions of the relevant profiles. Mix to Mix LRs were assigned using the FBI extended Caucasian allele frequencies with an LR threshold of zero. Extended output was also enabled.

Mix to Mix LRs (pairwise LRs and average LR) were then calculated within MS Excel across all loci. As part of the workup, the prior genotype probabilities and the posterior marginal



probabilities written to the STRmix<sup>TM</sup> extended output file were verified. The Mix to Mix *LR*s calculated manually and those assigned within STRmix<sup>TM</sup> are reproduced in the table below. Values have been provided to 10 significant figures.

Comparison	MS Excel	STRmix™
Single-source C1 vs Mix C1	0	0
Single-source C1 vs Mix C2	1.116575335E+29	1.116575335E29
Average LR	5.582876676E+28	5.582876676E28

The Mix to Mix *LR*s calculated in MS Excel match those assigned by STRmix<sup>™</sup> to 10 significant figures and therefore pass the acceptance criterion.

As an additional check, the *LR* from Previous function of STRmix<sup>TM</sup> was used to assign an *LR* for the known minor donor to the mixed profile. Under these circumstances (i.e. Mix to Mix comparison of an unambiguous single-source profile with a mixed profile), the pairwise Mix to Mix *LR*s are expected to be equal to the sub-sub-source *LR*s for corresponding contributor orders. The average Mix to Mix *LR* is expected to be equal to the sub-source *LR*. The results obtained are summarised in the table below and indicate that the Mix to Mix function is performing as expected.

Comparison	STRmix™ - Mix to Mix	STRmix <sup>™</sup> - <i>LR</i> from Previous
Single-source C1 vs Mix C1 /	0	0
LR Prev. contributor		
order 1		
Single-source C1 vs Mix C2 /	1.116575335E+29	1.116575335E29
LR Prev. contributor	· · · · · · · · · · · · · · · · · · ·	
order 2		
Average Mix to Mix LR /	5.582876676E+28	5.582876676E+28
LR Prev. sub-source		
LR		

#### <u>Test 2</u>

A second test was then performed whereby a two-person GlobalFiler<sup>M</sup> major/minor mixture was compared to the same profile with peak heights reduced by half. The STRmix<sup>M</sup> Mix to Mix function was used to compare prior deconvolutions of the relevant profiles. Mix to Mix *LR*s were assigned at two loci (D3S1358 and FGA) using the FBI extended Caucasian allele frequencies with an *LR* threshold of zero. Extended output was also enabled. The loci examined include instances of allelic dropout and/or drop-in.

As before, Mix to Mix *LR*s (pairwise *LR*s and average *LR*) were calculated within MS Excel. This included verification of the prior genotype frequencies and posterior marginal probabilities written to the STRmix<sup>TM</sup> extended output file. The Mix to Mix *LR*s calculated

STRmix™ Comparison MS Excel Mix 1, C1 vs Mix 2, C1 2.399680107E+03 2.399680107E+03 Mix 1. C1 vs Mix 2. C2 1.036066244E-02 1.036066244E-02 Mix 1, C2 vs Mix 2, C1 8.383930514E-03 8.383930514E-03 Mix 1, C2 vs Mix 2, C2 7.589432532E+04 7.589432532E+04 Average LR 1.957350604E+04 1.957350604E+04

manually and those assigned within STRmix<sup>™</sup> are reproduced in the table below. Values have been provided to 10 significant figures.

The Mix to Mix *LR*s calculated in MS Excel match those assigned by STRmix<sup>™</sup> to 10 significant figures and therefore pass the acceptance criterion.

#### Test 3

A third test was performed whereby a two-person unresolvable GlobalFiler™ mixture was compared with a three-person GlobalFiler<sup>™</sup> mixture with an approximate mixture ratio of 4:4:1. The two donors to the two-person mixture are the two major contributors to the threeperson mixture. The STRmix<sup>™</sup> Mix to Mix function was used to compare prior deconvolutions of the relevant profiles. Mix to Mix LRs were assigned using the FBI extended Caucasian allele frequencies with an LR threshold of zero. Extended output was also enabled.

Mix to Mix LRs (pairwise LRs and average LR) were calculated within MS Excel across all loci. The prior genotype frequencies and posterior marginal probabilities written to the STRmix<sup>™</sup> extended output file were used in these calculations. These values had previously been verified in the preceding two tests as described above. The Mix to Mix LRs calculated manually and those assigned within STRmix<sup>™</sup> are reproduced in the table below. Values have been provided to 10 significant figures.

Comparison	MS Excel	STRmix™
Mix 1, C1 vs Mix 2, C1	3.178939538E+19	3.178939538E+19
Mix 1, C1 vs Mix 2, C2	2.468184053E+19	2.468184053E+19
Mix 1, C1 vs Mix 2, C3	3.608969743E+08	3.608969743E+08
Mix 1, C2 vs Mix 2, C1	1.925156112E+17	1.925156112E+17
Mix 1, C2 vs Mix 2, C2	4.898637197E+17	4.898637197E+17
Mix 1, C2, vs Mix 2, C3	7.923178106E+07	7.923178106E+07
Average LR	9.525602539E+18	9.525602539E+18

The Mix to Mix *LR*s calculated in MS Excel match those assigned by STRmix<sup>™</sup> to 10 significant figures and therefore pass the acceptance criterion.

Test 4



Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

The performance of the Mix to Mix function when used to compare varNOC deconvolutions was then examined. The height reduced input file from Test 2 was deconvoluted assuming a range of 2-3 contributors. The STRmix<sup>™</sup> Mix to Mix function was then used to compare this interpretation with that of the unedited profile. Mix to Mix LRs were assigned using the FBI extended Caucasian allele frequencies with an LR threshold of zero. Extended output was also enabled.

Mix to Mix LRs (pairwise LRs and average LR) were calculated within MS Excel across all loci. The average contributor stratified LR was also calculated. The prior genotype frequencies and posterior marginal probabilities written to the STRmix<sup>™</sup> extended output file were used in these calculations. The Mix to Mix LRs calculated manually and those assigned within STRmix<sup>™</sup> are reproduced in the table below. Values have been provided to 10 significant figures.

Comparison	MS Excel	STRmix™
Mix 1, C1 vs Mix 2, C1	3.675025497E+40	3.675025497E+40
Mix 2 NoC = 2		
Mix 1, C1 vs Mix 2, C2	2.562159406E-15	2.562159406E-15
Mix 2 NoC = 2		
Mix 1, C2 vs Mix 2, C1	1.000722088E-22	1.000722088E-22
Mix 2 NoC = 2		
Mix 1, C2 vs Mix 2, C2	2.986561507E+23	2.986561507E+23
Mix 2 NoC = 2		
Average <i>LR</i> , Mix 2 NoC = 2	9.187563742E+39	9.187563742E+39
Mix 1, C1 vs Mix 2, C1	4.539045089E+40	4.539045089E+40
Mix 2 NoC = 3		
Mix 1, C1 vs Mix 2, C2	1.168455542E+08	1.168455542E+08
Mix 2 NoC = 3		
Mix 1, C1 vs Mix 2, C3	3.717317095E+04	3.717317095E+04
Mix 2 NoC = 3		
Mix 1, C2 vs Mix 2, C1	4.313139308E-27	4.313139308E-27
Mix 2 NoC = $3$		
Mix 1, C2 vs Mix 2, C2	7.203840318E+19	7.203840318E+19
Mix 2 NoC = $3$		
Mix 1, C2 vs Mix 2, C3	1.145037269E+13	1.145037269E+13
Mix 2 NoC = $3$		
Average <i>LR</i> , Mix 2 NoC = 3	7.565075148E+39	7.565075148E+39
Contributor stratified LR	7.565075148E+39	7.565075148E+39

The Mix to Mix *LR*s calculated in MS Excel match those assigned by STRmix<sup>™</sup> to 10 significant figures and therefore pass the acceptance criterion. In this example, the



contributor stratified LR closely aligned with the average Mix to Mix LR where three contributors were assumed to mix 2 (displayed in the second last row of the table above). This was due to the fact that the varNOC weights used in the assignment of the contributor stratified LR (i.e.  $Z_n$ ) heavily favoured an explanation of three contributors to this mixture ( $Z_n$ ) two contributors = 1.1731099947664E-20,  $Z_n$  three contributors = 1.0).

#### Test 5

As a final test, two varNOC deconvolutions were compared with each other using the STRmix<sup>™</sup> Mix to Mix function. Specifically, a single-source Identifiler<sup>™</sup> profile interpreted assuming a range of 1-2 contributors was compared with a three-person GlobalFiler™ mixture interpreted assuming a range of 2-3 contributors. To better demonstrate that the contributor stratified LR is being calculated correctly, the adjusted varNOC weights  $(Z_n)$  were modified to be equal to 0.5 for each value of N in both deconvolutions. Mix to Mix LRs were assigned using the FBI extended Caucasian allele frequencies with an LR threshold of zero. Extended output was also enabled.

Mix to Mix LRs (pairwise LRs and average LR) were calculated within MS Excel across all loci. The average contributor stratified LR was also calculated. The prior genotype frequencies and posterior marginal probabilities written to the STRmix™ extended output file were used in these calculations. The average Mix to Mix LRs and the average contributor stratified *LR* calculated manually and assigned within STRmix<sup>™</sup> are reproduced in the table below. Values have been provided to 10 significant figures.

Comparison	MS Excel	STRmix™
Average Mix to Mix LR	2.415697655E+08	2.415697655E+08
Left mix NoC = 1		
Right mix NoC = 2		
Average Mix to Mix LR	6.235550567E+07	6.235550567E+07
Left mix NoC = 1		
Right mix NoC = 3		
Average Mix to Mix LR	7.282741000E+07	7.282741000E+07
Left mix NoC = 2		
Right mix NoC = 2		
Average Mix to Mix LR	1.851961784E+07	1.851961784E+07
Left mix NoC = 2		
Right mix NoC = 3		
Contributor stratified LR	9.881807476E+07	9.881807476E+07

The Mix to Mix *LR*s calculated in MS Excel match those assigned by STRmix<sup>™</sup> to 10 significant figures and therefore pass the acceptance criterion. While not reproduced here, all pairwise Mix to Mix LRs calculated manually also matched the corresponding STRmix™ result to 10 significant figures.



# 4.18. Top-down approach

Acceptance criterion: Where full precision is output in STRmix, precision is expected to 10 sig fig. DNA amounts are given as a whole number and precision is expected to the full number given. Allele and stutter variances, degradation, and locus efficiencies are given to three decimal places and precision is expected to 3dp.

A three-person mixture was analysed using a top-down approach with Top N=2. Nineteen of the possible twenty steps were completed. The results from each step of the top-down approach were duplicated by manually running an interpretation and database search with the same settings (eg. the same seed, detection thresholds, Fst value and kit settings). The interpretation Results.txt and DBSearchResults.txt files from the manual runs where compared with their corresponding files in the top-down Steps results subfolder, using BeyondCompare. Where rounding is necessary, numerical values are displayed to at least 10 significant figures in both the Results.txt and DBSearchResults.txt files. The results for Step 5 of the top-down approach and the corresponding manual run are given in the table below. The detection threshold values used for each run are also displayed.

Parameter	Top-down approach (Step 5)	Manual run
Seed	807471	807471
Total iterations	2437743	2437743
Effective Sample Size	6951.289384	6951.289384
Average log(likelihood)	14.37478793	14.37478793
Gelman-Rubin convergence diagnostic	1.152482575	1.152482575
DNA Amounts: Contributor 1	287	287
DNA Amounts: Contributor 2	177	177
DNA Amounts (mode): Contributor 1	246	246
DNA Amounts (mode): Contributor 2	181	181
Mixture Proportions (%): Contributor 1	61.83933661	61.83933661
Mixture Proportions (%): Contributor 2	38.16066338	38.16066338
Mixture Proportions (%) (mode): Contributor 1	57.56386754	57.56386754
Mixture Proportions (%) (mode): Contributor 2	42.43613245	42.43613245

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

			Date of Issue: 29 September 2020
LSAE	Variance	0.03059442890	0.03059442890
Varia	nce	2.948	2.948
Back	Stutter Variance	11.342	11.342
Forwa	ard Stutter Variance	3.080	3.080
Degra	adation (rfu/bp), starting at 122.0bp: Contributor 1	0.805	0.805
Degra	adation (rfu/bp), starting at 122.0bp: Contributor 2	0.403	0.403
Degra	adation curve d values: Contributor 1	0.004	0.004
Degra	adation curve d values: Contributor 2	0.003	0.003
	D8S1179	71	71
	D21S11	116	116
	D7S820	115	115
	CSF1PO	89	89
	D3S1358	103	103
	TH01	109	109
	D13S317	164	164
	D16S539	132	132
	D2S1338	115	115
	D19S433	129	129
	vWA	69	69
	ТРОХ	187	187
(%)	D18S51	125	125
icies (	D5S818	132	132
Efficiencies (%)	FGA	109	109
ш	Suspect01A	2.712962496E-4	2.712962496E-4
£	Victim01A	0.002624427661	0.002624427661
urce <i>LR</i>	Suspect02A	1.080178660E8	1.080178660E8
n			

Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29	September 2020
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			Date of Issue: 29 September 2020
	Suspect02B	0.5046853411	0.5046853411
	Suspect02C	0.003291804394	0.003291804394
	Suspect02D	1.282176332E-4	1.282176332E-4
	Victim03A	0.01825567560	0.01825567560
	Consensual partner03	1.519103863E-4	1.519103863E-4
	Suspect03A	2.173796841E-4	2.173796841E-4
	Suspect03B	0.1215428090	0.1215428090
	Victim04A	0.009972502188	0.009972502188
	Suspect04A	0.009505451861	0.009505451861
	Suspect05A	0.001797692221	0.001797692221
	Suspect05B	2.317439023E-4	2.317439023E-4
	Suspect05C	0.002088474065	0.002088474065
	D8S1179	232	232
	D21S11	394	394
	D7S820	321	321
	CSF1PO	173	173
	D3S1358	317	317
	TH01	303	303
	D13S317	671	671
	D16S539	411	411
	D2S1338	299	299
	D19S433	659	659
	vWA	155	155
(î	ТРОХ	468	468
ds (rfi	D18S51	367	367
eshol	D5S818	578	578
on Thresholds (rfu)	FGA	351	351
ō			

The differences are 0 in the  $10^{th}$  sig fig. Hence, these tests satisfy the acceptance criteria.

# 4.19. $H_d$ true tester

Acceptance criterion: Precision is expected to 10 sig fig.

A one locus single source profile with ambiguity and one locus two person mixed DNA profile were analysed using the  $H_d$  True Tester module. 10,000 profiles were generated and importance sampling enabled. The *LR*, bias, effective iterations and average *LR* were calculated. The results are given in the table below.

Single source example, where	$\hat{f_{12}} = 0.314142461964039$

D16S539, <i>Sy</i>	Weight, <i>w</i> <sub>y</sub>	Fraction in database	$\Pr(S_{y})$
12,12	0.8264563264180106	0.8234	0.0986854864088275
12,Q	0.173543673581989	0.1766	0.430913951110422

STRmix™	10		
Database profile	LR	$b_{y} = \frac{\Pr(S_{y})}{w_{y}}$ Bias	Effective iterations
12,12	5.286220323	0.1194079871	0.5396468233
12,Q	1.110028521	2.483028866	11.22168350
Average LR		# effective iterations	
$\overline{LR} = \frac{1}{y} \sum_{y} LR_{y} b_{y}$	1.006494742	$I = \frac{\sum_{y} LR_{y}}{\overline{LR}}$	45193.52819
Excel			
Database profile	LR	$b_{y} = \frac{\Pr(S_{y})}{w_{y}}$ Bias	Effective iterations
12,12	5.286220323	0.1194079871	0.5396468233
12,Q	1.110028521	2.483028866	11.22168350
Average LR $\overline{LR} = \frac{1}{y} \sum_{y} LR_{y} b_{y}$	1.006494742	# effective iterations $I = \frac{\sum_{y} LR_{y}}{\overline{LR}}$	45193.52819



The differences are 0 in the 10<sup>th</sup> sig fig. Hence, these tests satisfy the acceptance criteria.

#### Mixture example

D18S51, <i>S</i> <sub>y</sub>	Weight, <i>w<sub>y</sub></i>
16,20 and 12,16	0.425666776424102
16,20 and 12,20	0.224604873230161
16,20 and 12,12	0.19250082318077
16,20 and 12,19	0.112734606519591
16,20 and 12,15	0.0363845900559762
16,20 and 12,Q	0.00810833058939743

STRmix™			
Database profile	LR	$b_{y} = \frac{\Pr(S_{y})}{w_{y}}$ Bias	Effective iterations
[16,20]	94.32535321	0.01060160355	0.5370423171
[12,16]	10.15810311	0.1564660955	7.926057040
[12,20]	5.359966032	0.03652037839	1.850002080
[12,19]	2.690296309	0.1618839739	8.200508917
[12,12]	4.593835648	0.1338475101	6.780274006
[12,15]	0.8682811018	2.123911358	107.5903538
Average <i>LR</i> $\overline{LR} = \frac{1}{y} \sum_{y} LR_{y} b_{y}$	1.004951937	$I = \frac{\sum_{y} LR_{y}}{\frac{1}{LR}}$	506567.0625
Excel			
	LR	Bias	Effective iterations
[16,20]	94.32535321	0.01060160355	0.5370423171
[12,16]	10.15810311	0.1564660955	7.926057040
[12,20]	5.359966032	0.03652037839	1.850002080
[12,19]	2.690296309	0.1618839739	8.200508917
[12,12]	4.593835648	0.1338475101	6.780274006
[12,15]	0.8682811018	2.123911358	107.5903538
Average LR:	1.004951937	# effective iterations	506567.0625
			•

STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020 These numbers are the same to the 10<sup>th</sup> sig fig. Hence, these tests satisfy the acceptance criteria.

#### 4.20. Model Maker

Acceptance criterion: MM results output to 15 sig fig in STRmix<sup>™</sup> except per sample template and LSAE values that are rounded to the nearest whole number. Precision is expected to 10 sig fig.

Model Maker works by using a component-wise MCMC. In component 1, each profile has its mass parameters optimised, using a stable gamma distribution when sampling allele and stutter constants and a stable exponential distribution when sampling the LSAE variance constant. In component 2, the mass parameters for each profile are held constant and the hyper-parameters for each distribution (for allele, each stutter type, and LSAE variance) are varied. A hyper-parameter is a parameter of a prior distribution. Within Model Maker, the hyper-parameters relate to the shape and scale parameters within the gamma distribution for allele and stutter and the mean of the exponential LSAE variance. The likelihood of the total dataset is recalculated at each iteration.

Components are 1000 accepts long and they cycle through a number of times depending on the user input number of accepts (default = 100,000). For example, if 100,000 accepts are chosen then there will be 100 cycles of the two components, if 200,000 is chosen there will be 200 cycles.

Each MCMC is a Gaussian walk MCMC with step-sizes as per a normal STRmix<sup>™</sup> calculation. This way they adjust in larger or smaller increments as the hyper-parameter becomes larger or smaller. In doing this the process has some memory, but doesn't require specific settings to do with step-sizes.

The final gamma distributions chosen are the set at the posterior mode of the posterior sample within the last 25% of the Model Maker iterations i.e. the set that gave the highest likelihood (versus being the last or an average value).

The extended output for two Model Maker runs was analysed in Excel. A summary of the component cycle log(likelihood) values from the report is given in the table below.

Component cycle	STRmix	Excel
1	-79.6436652212192	-79.6436652212192
2	-79.6326367530189	-79.6326367530189
3	-78.3041519933175	-78.3041519933175
4	-79.3585682507529	-79.3585682507529
5	-86.2530742186945	-86.2530742186945
6	-82.7768477687874	-82.7768477687874
7	-80.8892117226486	-80.8892117226486

8	-87.2366956340459	-87.2366956340459
9	-85.7081860522184	-85.7081860522184
10	-85.6989939063051	-85.6989939063051

These numbers are the same to 10 sig fig. Hence, they pass the acceptance criterion.

A summary of the allele, stutter, and LSAE variances are in the table below.

Туре	Alpha - STRmix	Beta – STRmix	Alpha - Excel	Beta – Excel
Allele	3.19830509012194	5.61810201194136	3.19830509012194	5.61810201194136
Back Stutter	3.33346506249546	4.25748402418060	3.33346506249546	4.25748402418060
Forward Stutter	3.25503804759332	6.92949977966533	3.25503804759332	6.92949977966533
LSAE variance	98.8766115735246	0.0101136151824579	98.8766115735246	0.010113615182457

These numbers are the same to 10 sig fig. Hence, they pass the acceptance criterion. A summary of the mass parameters for one profile are given in the table below (5 LSAE only).

	Template	Degradation	Locus 1	Locus 2	Locus 3	Locus 4	Locus 5
STRmix	1264	6.01053343431042E-04	100%	117%	83%	118%	76%
Excel	1264	6.01053343431041E-04	100%	117%	83%	118%	76%

These numbers are the same to 10 sig fig. Hence, they pass the acceptance criterion.

#### 4.21. Variable number of contributors function

Note that the developmental validation of the range of contributors (or varNOC) function within STRmix<sup>TM</sup> has been restricted to a difference in *n* of only 1.

A varNOC interpretation starts with the deconvolution of the profile under the differing assignments of *n*. This deconvolution follows the same MCMC process as a typical deconvolution with the generation of genotype combinations within pre burn-in, followed by the determined number of burn-in accepts, and then post burn-in accepts. Setting the seed within a single *n* deconvolution will return the same deconvolution results as the *n* component of a varNOC deconvolution with the same seed. For varNOC, the seed is conserved between the different *n* interpretations to facilitate this check.

The primary output from varNOC are the contributor number probabilities i.e. weights,  $Z_n$ , used within the calculation of the varNOC *LR*. The validation of these outputs, following the

algorithms described in the STRmix™ User's Manual was undertaken across nine interpretations, including but not limited to examples using replicate PCR inputs generated in the same or different kits, where drop-in and dropout were variously modelled, and where autocontinuation was triggered based on GR value.

Described below is an example of the calculations replicated from the extended output from a profile analysed using replicates and (unless otherwise specified) a range of contributors (n=1 and 2), with two chains, each containing 5000 burn-in and post burn-in accepts, and with hyper-rectangle set to contain 2.5 percent of the total MCMC accepts.

# 4.21.1. Hyper-rectangle

For each chain the hyper-rectangle is drawn around the MCMC iteration with the highest posterior probability. In an extended output the posterior probabilities (thinned to every 10th accept) are written to the file labelled in part "Max Posterior Probability". These values are the inverse log of the total MCMC probability for each accepted MCMC iteration recalculated to sum across all accepted genotype combinations. For each *n* the accepted genotype combinations per locus are listed in the corresponding GenotypePdf file.

The following steps describe a modified Weinberg algorithm<sup>3</sup> for the calculation of a hyperrectangle from an extended output of the varNOC replicates example described above. This is repeated for each chain and each n within the contributor range. For varNOC multikit scenarios, the kit efficiency parameters are also included in these calculations. The method to determine the hyper-rectangle is broken up into 6 steps that are describe below.

Step 1. Acceptance criterion: Precision is expected to 10 significant figures to the maximum in the extended output "1C Max Posterior Probability 1".

The posterior sample with the highest likelihood (defined as  $\theta_{neak}$ ) from the entire posterior

MCMC sample ( $\Omega$ ) of chain 1 was identified from the extended output "1C Max Posterior Probability 1". For n=1 chain 1, the maximum was identified as 7.033393771E-4 for accept 1271. From the log, this was confirmed as the "Maximum posterior probability of 7.0333937719E-4 in chain 1 at position 1271".

These tests satisfy the acceptance criteria.

Step 2. Acceptance criteria: The squared distances for each (11) mass parameter values and their sum are output per iteration to 15 sig fig in the extended output file labelled in part "Q0 distances". Parameter values for theta peak, minimum and maximum mass parameters are output to the Interpretation.txt to 15 sig fig. Precision is expected to 10 sig fig.

The squared distance for each peak variance parameter, LSAE variance parameter, replicate efficiency, template, degradation and LSAE parameter within the 5,000 post burn-in accepts from the corresponding parameter values at  $\theta_{neak}$  identified in step 1 were

<sup>&</sup>lt;sup>3</sup> Weinberg, M.D. Yoon, I. Katz, N A remarkably simple and accurate method for computing the Bayes Factor from a Markov chain Monte Carlo Simulation of the Posterior Distribution in high dimension. arXiv:1301.3156v1 [astro-ph.IM] 14 Jan 2013



Issued by Institute of Environmental Science and Research Limited

Date of Issue: 29 September 2020

calculated. These squared distances (and their sum) are printed to the extended output "Q0\_distances". The parameter values for  $\theta_{peak}$ , minimum mass parameters and maximum mass parameters are printed to the log. The values for chain 1 *n*=1 are replicated below:

STRmix™			
Parameter	<b>θ</b> <sub>peak</sub>	Min value	Max value
Template	146.0227269	85.17521810	194.9547979
Degradation	0.005404250766	0.004763308227	0.009999411039
$\sigma_A^2$	0.005413234625	0.003800381771	0.04658405605
LSAE D3S1358	1.087506025	0.6375913459	1.783865504
LSAE vWA	1.041038174	0.5849071934	1.461332569
LSAE CSF1PO	1.011147176	0.6325915338	1.251410747
PCR1	1.979409485	0.8465530441	3.304798482
PCR2	0.50520117611	0.3025903107	1.181260887
<i>C</i> <sup>2</sup>	7.247820263	6.479049025	7.639948860
$k_{a-1}^2$	7.669425682	6.292885184	9.031396254
$k_{a+1}^2$	5.702647873	5.023280953	6.139612456
Excel			
Parameter		Min value	Max value
Template		85.17521810	194.9547979
Degradation		0.004763308227	0.009999411039
$\sigma_{\scriptscriptstyle A}^2$		0.003800381771	0.04658405605
LSAE D3S1358		0.6375913459	1.783865504
LSAE vWA		0.5849071934	1.461332569
LSAE CSF1PO		0.6325915338	1.251410747
PCR1		0.8465530441	3.304798482



STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

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0.3025903107	1.181260887
6.479049025	7.639948860
6.292885184	9.031396254
5.023280953	6.139612456
	0.3025903107 6.479049025 6.292885184

These numbers are the same to the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

Step 3. Acceptance criterion: The top 125<sup>th</sup> accept is expected to be the identical iteration detailed in the Interpretation.txt output. The initial sample distance (d bar) is output to 16 sig fig in the Interpretation.txt. Precision is expected to 10 sig fig.

The summed distances were sorted from lowest to highest and the top 125 accepts (2.5 percent of the total MCMC accepts for chain 1 n=1 for this example) were identified. The top 125<sup>th</sup> accept was the 317<sup>th</sup> iteration. This was confirmed within the log for this sample; "125 accepted samples resulted in 317 MCMC iterations".

Excel	STRmix	
	-	
0.3354336864	0.3354336864	
0.000400004	0.000400004	
		· · · · · · · · · · · · · · · · · · ·

These numbers are the same to the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

Step 4. Acceptance criteria: The updated squared distances for each mass parameter values and their sum are output per iteration to 15 sig fig in the extended output file labelled in part "Q1\_distances". Precision is expected to 10 sig fig.

The root mean variance of each of the parameters for each iteration of the post burn-in accepts was calculated. This is the square root of the summed differences of the parameter to the corresponding  $\theta_{peak}$  parameter value identified in step 1, divided by the total number of iterations represented by the top 125 accepts for this chain. The squared distances were then recalculated as in step 2 using the root mean variance. These updated values and their sum were confirmed by comparison to extended output "Q1\_distances". The values for chain 1 *n*=1 are replicated below:

Parameter	Excel	STRmix
Template	7.374405137	7.374405137
Degradation	9.864714794	9.864714794

		Date of 135de. 29 Deptemb
$\sigma_{\scriptscriptstyle A}^2$	2.158712793	2.158712793
LSAE D3S1358	36.90325530	36.90325530
LSAE vWA	18.93335569	18.93335569
LSAE CSF1PO	0.0002162115253	0.0002162115253
PCR1	0.04988464719	0.04988464719
PCR2	0.05493124769	0.05493124769
<i>C</i> <sup>2</sup>	38.96944339	38.96944339
k <sub>a-1</sub>	121.9621485	121.9621485
$k_{a+1}^2$	0.4708947576	0.4708947576
Total	236.7419624	236.7419624

These numbers are the same to the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

Step 5. Acceptance criterion: The revised top 125th accept is expected to be the identical iteration detailed in the Interpretation.txt output. The updated sample distance (d bar) is output to 17 sig fig in the Interpretation.txt. Precision is expected to 10 sig fig.

The updated summed distances were sorted from lowest to highest and the top 125 accepts identified. The top 125<sup>th</sup> accept was identified as the 362<sup>nd</sup> iteration. This was confirmed within the log for this sample; "125 accepted samples resulted in 362 MCMC iterations".

Excel	STRmix
3.817499653	3.817499653

These numbers are the same to the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

Step 6. Acceptance criteria: The extrema of each mass parameter within the hyperrectangle are output to 15 sig fig in the Interpretation.txt and the extended output file labelled in part "1C\_HR\_shape\_1". Precision is expected to 10 sig fig.

The hyper-rectangle is drawn around the top 125 accepted iterations. The extrema of each parameter within these iterations were determined. These values are written to the extended output file named in part 1C\_HR\_shape\_1. The volume, *V*, of the hyper-rectangle is calculated by taking the product of the range (difference) for each parameter. *V* is replicated for n=1 chain 1 below:

STRmix™		
Parameter	Minimum	Maximum
Template	101.1109928	148.7622312
Degradation	0.005175907701	0.006176379061
$\sigma_A^2$	0.0038073628	0.01476719248
LSAE D3S1358	1.034461994	1.169056219
LSAE vWA	0.9725499324	1.120733226
LSAE CSF1PO	1.000951381	1.180689626
PCR1	1.699550678	2.246809917
PCR2	0.4450754789	0.5883908099
<b>C</b> <sup>2</sup>	7.212786887	7.360963129
$k_{a-1}^2$	7.555787759	7.970356417
$k_{a+1}^2$	5.584899986	5.753800267
Excel		
Parameter	Minimum	Maximum
Template	101.1109928	148.7622312
Degradation	0.005175907701	0.006176379061
$\sigma_{\scriptscriptstyle A}^2$	0.0038073628	0.01476719248
LSAE D3S1358	1.034461994	1.169056219
LSAE vWA	0.9725499324	1.120733226
LSAE CSF1PO	1.000951381	1.180689626
PCR1	1.699550678	2.246809917
PCR2	0.4450754789	0.5883908099
<b>C</b> <sup>2</sup>	7.212786887	7.360963129
$k_{a-1}^2$	7.555787759	7.970356417
$k_{a+1}^2$	5.584899986	5.753800267

These numbers are the same in the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

# 4.21.2. MC average

Acceptance criteria: The per locus dropout frequencies are output to 15 sig fig in the Interpretation.txt file.

The per locus genotype set probabilities, mass parameter penalties, and log(profile probability) values per iteration are output to 15 sig fig in the extended output file labelled in part "1C Naive MC".

Under each *n* the per chain MC average and Unadjusted MC average values are output to 15 sig fig in the Interpretation.txt. Precision is expected to 10 sig fig.

For each chain and each n, once the HR is defined, a naïve MC sampling of the parameters is conducted drawing 10,000 new values uniformly from the range enveloped by the hyperrectangle. For each MC iteration, the posterior likelihoods, log(p), are recalculated for all peaks, for each proposed genotype set. The inverse log of the sum of the log(p) values are each multiplied by the probability of the corresponding genotype set,  $Pr(S_i)$ , calculated  $(F_{ST}=0)$  using the posterior mean allele frequencies for the chosen Population for Range and incorporating Pr(C) where drop-in is proposed. Where dropout is proposed the per locus dropout frequencies are obtained by subtracting from 1 the sum of the posterior mean allele



frequencies for all alleles present in the posterior burn-in MCMC sample at that locus under the corresponding *n*. These dropout frequencies are written to the log for each *n*. The Locus Probability value per locus is then obtained by summing across the genotype combinations.

The log(product) of the locus probabilities is added to the sum of the mass parameter penalties for all peak variances, LSAEs and LSAE variances and a combined template/degradation penalty across all contributors to obtain the unadjusted log(profile probability) value. These penalties can be calculated using the formulae in the table below, where *x* is the sampled parameter value for the MC iteration, and we obtain from the kit file  $\alpha$ , $\beta$  describing the corresponding variance's prior gamma distribution, the mean LSAE variance  $1/\lambda$ , saturation and deg max.

Parameter type	Penalty formula
Peak variance	=log(GAMMA.DIST( $x, \alpha, \beta, 0$ )
LSAE	=log(product(L))
	Where L is calculated per locus by
	=NORM.DIST(log(x),0,SQRT(y),0)
	And where y is the LSAE variance value sampled for the MC iteration
Template and degradation	=log(product(C))
aogradation	Where C is calculated as per contributor by
	(1/minimum saturation per kit) <sup>n</sup> for template, and
	(1/minimum deg max per kit) <sup>n</sup> for degradation
LSAE variance	=EXPON.DIST(x,λ,0)
	Where $\lambda$ is the reciprocal of the mean LSAE variance

The Adjusted log(profile probability) value is calculated by adding the log(product) of the per locus adjustments to the sum of the penalties. These values are replicated below for chain 1 n=1 Iteration 1 for a 3-locus example where generalised stutter was enabled and a double drop-in event was proposed (modelled U[0,0] with frequency = 0.0029) at SE33. The chosen Population for Range was FBI\_extended\_Cauc.

varNOC extended output value	STRmix™	Excel
Pr( <i>S<sub>j</sub></i> ) SE33 [29.2,30]	4.264886417E-05	4.264886417E-05

Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

		Date of issue. 29 September 2
Pr(S <sub>j</sub> ) D22S1045 [15,16]	2.298011632E-01	2.298011632E-01
Pr( <i>S<sub>j</sub></i> ) FGA [21,23]	5.553288257E-02	5.553288257E-02
Locus probability SE33	1.722603204E-09	1.722603204E-09
Locus probability D22S1045	1.931449339E+02	1.931449339E+02
Locus probability FGA	4.373730062E+01	4.373730062E+01
LSAE penalty	0.6703954923	0.6703954923
LSAE variance penalty	1.641603480	1.641603480
Allele variance penalty	-1.140829899	-1.140829899
Back Stutter variance penalty	-1.910237172	-1.910237172
Forward Stutter variance penalty	-0.8099411461	-0.8099411461
Template and degradation penalty	-2.477121254	-2.477121254
log(profile probability)	-2.599019861	-2.599019861
log(profile probability) unadjusted	-8.863209951	-8.863209951

These numbers are the same in the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

The average of the inverse log of each of the 10,000 calculations for both the adjusted and unadjusted profile probabilities are printed to the Log as the "MC average" and "Unadjusted MC average" per chain and *n*. These were replicated as shown below.

STRmix™						
N=1	Chain 1	Chain 2				
Marginal Likelihood	7.042094814E-12	1.445465161E-11				
Unadjusted Marginal						
Likelihood	5.421674180E-15	1.290061869E-14				
N=2	Chain 1	Chain 2				

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Date of Issue: 29 September 2020

		Date of Issue. 29 Septe
Marginal Likelihood	4.591872433E-13	4.369723161E-13
Unadjusted Marginal		
Likelihood	2.744245475E-18	2.743538057E-18
Excel	1	<u> </u>
N=1	Chain 1	Chain 2
Marginal Likelihood	7.042094814E-12	1.445465161E-11
Unadjusted Marginal		
Likelihood	5.421674180E-15	1.290061869E-14
N=2	Chain 1	Chain 2
Marginal Likelihood	4.591872433E-13	4.369723161E-13
Unadjusted Marginal		
Likelihood	2.744245475E-18	2.743538057E-18

These numbers are the same in the 10<sup>th</sup> sig fig. Hence they pass the acceptance criterion.

# 4.21.3. F(Omega)

Acceptance Criterion: F(omega) is output per chain to 16 sig fig in the Interpretation.txt file. Precision is expected to 10 sig fig.

For each chain and *n*,  $\Omega$  (omega) is the posterior burn-in sample from the MCMC deconvolution for that chains. The number of total iterations is  $|\Omega|$ .  $\Omega_s$  is a subset of the sample that falls within the HR and  $|\Omega_s|$  is the count of iterations within this subset.  $|\Omega_s|$  is determined by counting all iterations (including rejects) of the extended PostBurninResults for the given chain and *n* where the parameter values fall between the corresponding minimum and maximum determined in step 6 of the HR process above. For chain 1 *n*=1,

$$|\Omega_s|$$
=686 and  $|\Omega|$ =9,857.  $F(\Omega_s) = \frac{|\Omega_s|}{|\Omega|} = 0.06959521152$  (Excel) and 0.06959521152 in

the extended output.

These numbers are the same to 10 sig fig and hence these tests satisfy the acceptance criteria.

#### 4.21.4. Marginal likelihoods

Acceptance Criterion: The per chain marginal likelihoods and normalized values are output to 16 sig fig in the Interpretation.txt file. They also appear as the Transdimensional Parameters in the Results.txt file. Precision is expected to 10 sig fig.

The marginal likelihoods are calculated using the following formula:

$$p(\boldsymbol{O}|N_n) = \frac{V}{F(\Omega_s)Y} \sum_{i=1}^{Y} \sum_{j=1}^{Y} p(\boldsymbol{O} \mid N_n, \boldsymbol{M}_n^{(i)}, \boldsymbol{S}_j) p(\boldsymbol{M}_n^{(i)} \mid N_n) \Pr(\boldsymbol{S}_j \mid N_n)$$

where *V* is the volume of the HR described above,  $F(\Omega_s)$  is also described above, *Y* is the number of MC iterations performed, and the final term is the summation across *Y* of the corresponding profile probability. It is this summation divided by *Y* that gives the MC average per chain described above. Within the Interpretation log they are reported as "marginal likelihood" and "unadjusted marginal likelihood" for each chain.

These values are averaged across chains then normalised across *n*. The normalised marginal likelihood using the adjusted profile probabilities is  $Z_n$  used within the calculation of the varNOC *LR*. The normalised marginal likelihood using the unadjusted profile probabilities is the p(N | O). Both sets of normalised values appear in the Transdimensional parameters section of the raw Results.txt file. The normalised values from the STRmix<sup>TM</sup> log were replicated in Excel and are given for the example described here in the table below:

	STRmix™	Excel
N=1	Normalised Average	Normalised Average
Marginal Likelihood	0.9599802026	0.9599802026
Unadjusted Marginal Likelihood	0.9997005756	0.9997005756
N=2	Normalised Average	Normalised Average
Marginal Likelihood	0.04001979733	0.04001979733
Unadjusted Marginal Likelihood	0.0002994243349	0.0002994243349

These numbers are the same in the 10<sup>th</sup> sig fig . Hence these tests satisfy the acceptance criterion.

# 4.22. varNOC likelihood ratios

#### 4.22.1. varNOC sub-source LRS

Acceptance criterion: Precision is expected in Excel to 10 sig fig.

There are three methods for assigning a Likelihood Ratio for a varNOC interpretation; a stratified *LR*, a Maximum Likelihood Estimate (MLE) *LR* and an *LR* using user defined *N* under  $H_p$  and user defined *N* under  $H_d$ .

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The stratified likelihood ratio treats N as a nuisance parameter and integrates N out by stratifying the LR across the variable contributor numbers assigned in the deconvolution. The MLE method of assigning the LR uses the most probable number of contributors under each hypothesis i.e. the N that produces the maximum posterior probability of the epg under  $H_n$ 

and under  $H_d$ . The user defined assigns an *LR* using the user selected *N* under  $H_p$  and the user selected *N* under  $H_d$ . Note that the choice between *LR*s only affects the values of  $Pr(N_p | H_x)$  terms in the calculation of the *LR*.

As part of the V2.8 validation, 127 varNOC *LR*s were replicated in Excel. These included single population and stratified population, mixed DNA profiles, sub-sub source and sub-source *LR*s, unrelated, related, and unified propositions, and the HPD.

# Stratified LR replication

The stratified *LR*s are assigned using the formula:

$$LR = \frac{\sum_{n} Z_{n} \sum_{j} w_{j} \operatorname{Pr}(S_{j} | N_{n}, H_{p})}{\sum_{n} Z_{n} \sum_{j} w_{j} \operatorname{Pr}(S_{j} | N_{n}, H_{d})}$$

Where  $Z_n$  is the adjusted probability of *n* given the profile and is available from the deconvolution results.

The sub-source LR uses the average of all  $Pr(E | H_p)$  across contributor orders and the

average of all  $Pr(E | H_d)$  across contributor orders for each *n* assigned, and the sub-sub-

source *LR* uses the maximum  $Pr(E | H_p)$  and  $Pr(E | H_d)$  across the contributor orders and

for each *n* assigned. These probabilities are then weighted using the adjusted probability of *n* given the profile. The replication of the sub-source stratified varNOC unrelated and related *LR*s for profile interpreted assuming 2 and 3 contributors is given in the table below. The FBI extended African American allele frequencies were applied with a theta value of 3%. The population was set to 1 million and the average number of children was set to 4 (these values are used to determine the priors on the relationship type in  $H_d$  used in the unified *LR*).

LR type	Relationship	Excel	STRmix
Stratified	Sibling	0.256409647	0.256409647
	Parent child	0.216130973	0.216130973
	Half sibs, etc	1.090975980	1.090975980
	Cousin	4.243243038	4.243243038
	Unrelated	33.58608354	33.58608354
	Unified	33.52809658	33.52809658

These numbers match and hence these tests satisfy the acceptance criteria.

# Maximum Likelihood Estimate LR replication

The MLE *LR*s are replicated in Excel using the formula:

$$LR = \frac{Z_n \sum_{j} w_j \operatorname{Pr}(S_j | N_n, H_p)}{Z_{n'} \sum_{j'} w_{j'} \operatorname{Pr}(S_{j'} | N_{n'}, H_d)}$$

Where  $Z_n$  is the adjusted probability of *n* given the profile available from the deconvolution results and where *n* and *n*' are the number of contributors that produce the maximum posterior probability of the profile under  $H_p$  and  $H_d$ , respectively.

For the GlobalFiler<sup>TM</sup> example above, the *n* that produced the maximum posterior probability of the profile under  $H_p$  and  $H_d$  for the sub-source *LR* was calculated by taking the average  $Pr(E | H_p)$  and  $Pr(E | H_d)$  across the contributor orders for each *n* assigned and then weighting them using the adjusted probability *n* given the profile. In this example, *n* and *n*' are both 2.

The replication of the sub-source MLE varNOC unrelated and related *LR*s for the same profile as above is given in the table below. The FBI extended African American allele frequencies were applied with a theta value of 3%. The population was set to 1 million and the average number of children was set to 4 (these values are used to determine the priors on the relationship type in  $H_d$  used in the unified *LR*).

LR type	Relationship	Excel	STRmix
MLE	Sibling	0.25640937	7 0.256409377
	Parent child	0.21613074	7 0.216130747
	Half sibs, etc	1.09097489	6 1.090974896
	Cousin	4.24323915	2 4.243239152
	Unrelated	33.5860606	5 33.58606065
	Unified	33.5280737	1 33.52807371

These numbers match and hence these tests pass the acceptance criterion

#### User defined n LR replication

User defined sub-source *LRs* and sub-sub-source *LRs* were generated from the example above using the *LR* from previous function, and enabling the extended output.

The LRs were replicated in Excel using the formula:

$$LR = \frac{Z_n \sum_{j} w_j \operatorname{Pr}(S_j | N_n, H_p)}{Z_{n'} \sum_{j'} w_{j'} \operatorname{Pr}(S_{j'} | N_{n'}, H_d)}$$

Where  $Z_n$  is the adjusted probability of *n* given the profile available from the deconvolution results and where *n* and *n'* are user defined.

The sub-source *LR* uses the average of all  $Pr(E | H_p)$  across contributor orders and the average of all  $Pr(E | H_d)$  across contributor orders for the user defined *n*, and the sub-sub-

source *LR* uses the maximum  $Pr(E | H_p)$  and  $Pr(E | H_d)$  across the contributor orders for the user defined *n*. These probabilities were then weighted using the adjusted probability of *n* given the profile given the user defined *n*.

The replication of the point estimate sub-source *LR* assuming under  $H_p$  N=3 and under  $H_d$  N=2 for the same profile as above, are in the table below. The FBI extended African American allele frequencies were applied with a theta value of 3%. The population was set to 1 million and the average number of children was set to 4 (these values are used to determine the priors on the relationship type in  $H_d$  used in the unified *LR*).

LR type	Relationship	Excel	STRmix	
User defined	Sibling	3.15085386 × 10 <sup>-07</sup>	3.15085386 × 10 <sup>-07</sup>	
$H_{\rho}=3$	Parent child	2.65589507 × 10 <sup>-07</sup>	2.65589507 × 10 <sup>-07</sup>	
<i>H</i> <sub>p</sub> =2	Half sibs, etc	1.34063055 × 10 <sup>-06</sup>	1.34063055 × 10 <sup>-06</sup>	
	Cousin	5.21425020 × 10 <sup>-06</sup>	5.21425020 × 10 <sup>-06</sup>	
	Unrelated	4.12718014 × 10 <sup>-05</sup>	4.12718014 × 10 <sup>-05</sup>	
	Unified	4.12005448 × 10 <sup>-05</sup>	4.12005448 × 10 <sup>-05</sup>	

These numbers match and hence these tests pass the acceptance criterion.

# Population stratified varNOC LR replication

The population stratified; varNOC stratified, MLE and user defined n=3 under  $H_p$  and n=2 under  $H_d$ , sub-source, point estimate *LR* for the same profile where the population proportion was set to 0.7 for the Caucasian population, 0.3 for the African American population and the African American population size was set to 1000000 with four children while the Caucasian population was kept at 0 children, are below:

LR type	Relationship			
		Excel	STRmix	
Stratified	Sibling	1.592347924 × 10 <sup>-01</sup>	1.592347924 × 10 <sup>-01</sup>	
	Parent child	1.130796131 × 10 <sup>-01</sup>	1.130796131 × 10 <sup>-01</sup>	
	Half sibs, etc	5.899433127 × 10 <sup>-01</sup>	5.899433127 × 10 <sup>-01</sup>	
	Cousin	2.299525972	2.299525972	
	Unrelated	19.40778646	19.40778646	
	Unified	19.40770933	19.40770933	
MLE	Sibling	1.592345403 × 10 <sup>-01</sup>	1.592345403 × 10 <sup>-01</sup>	
	Parent child	1.130794349 × 10 <sup>-01</sup>	1.130794349 × 10 <sup>-01</sup>	
	Half sibs, etc	5.899424131 × 10 <sup>-01</sup>	5.899424131 × 10 <sup>-01</sup>	
	Cousin	2.299522654	2.299522654	
	Unrelated	19.40776328	19.40776328	
	Unified	19.40768615	19.40768615	
User defined	Sibling	2.818009490 × 10 <sup>-07</sup>	2.818009490 × 10 <sup>-07</sup>	
	Parent child	2.001192204 × 10 <sup>-07</sup>	2.001192204 × 10 <sup>-07</sup>	
	Half sibs, etc	1.044034363 × 10 <sup>-06</sup>	1.044034363 × 10 <sup>-06</sup>	
	Cousin	4.069516983 × 10 <sup>-06</sup>	4.069516983 × 10 <sup>-06</sup>	
	Unrelated	3.434635539 × 10 <sup>-05</sup>	3.434635539 × 10 <sup>-05</sup>	
	Unified	3.434621889 × 10 <sup>-05</sup>	3.434621889 × 10 <sup>-05</sup>	

These numbers macth and hence these tests pass the acceptance criterion.

#### 4.22.2. varNOC HPD *LR*

Precision is expected to 10 sig fig.

The HPD *LR* was replicated for all of the the varNOC scenarios described previously. The sub-source population stratified varNOC *LR*s are provided in the table below:

LR type	Relationshi			
	p	Excel	STRmix	
Stratified	Sibling	8.5677353428E-02	8.5677353428E-02	
	Parent child	5.3526548635E-02	5.3526548635E-02	
	Half sibs, etc	2.6627819890E-01	2.6627819890E-01	
	Cousin	9.5301325797E-01	9.5301325797E-01	
	Unrelated	6.5452395741E+00	6.5452395741E+00	
	Unified	6.5452357799E+00	6.5452357799E+00	
MLE	Sibling	8.5677045386E-02	8.5677045386E-02	
	Parent child	5.3526484692E-02	5.3526484692E-02	
	Half sibs, etc	2.6627740020E-01	2.6627740020E-01	
	Cousin	9.5300983557E-01	9.5300983557E-01	
	Unrelated	6.5452185896E+00	6.5452185896E+00	
	Unified	6.5452147954E+00	6.5452147954E+00	
User selected	Sibling	1.3439858261E-07	1.3439858261E-07	
	Parent child	9.4744093553E-08	9.4744093553E-08	
	Half sibs, etc	5.1023167679E-07	5.1023167679E-07	
	Cousin	1.8708679380E-06	1.8708679380E-06	
	Unrelated	1.3665088039E-05	1.3665088039E-05	
	Unified	1.3665083025E-05	1.3665083025E-05	

These numbers match and hence these tests pass the acceptance criterion.

#### 4.23. CODIS report

Acceptance criterion: alleles within the xml file and CODIS summary report as expected based on the settings logic.

The STRmix<sup>™</sup> CODIS report logic has been tested using 40 single source and mixed profiles with respect to the correct alleles being present on the report. This included changing the Genotype Weight Threshold (default 0.99), Max alleles (default 4), and the

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Date of Issue: 29 September 2020

option to force a single genotype if the threshold is reached. A three-person mixture was interpreted in STRmix<sup>™</sup> and CODIS reports run under a range of different settings. A summary of the component interpretation for contributor one (0.65 mixture proportion) for three loci is given below.

Locus	Genotype	Weight			
D3S1358	14, 16	97.78%			
	15, 16	1.90%			
	14, 15	0.33%			
D1S1656	11, 17	99.97%			
	17, 17	0.03%			
D2S441	10, 14	99.62%			
	11, 14	0.27%			
	10, 11	0.11%			

The alleles present within the CODIS file and CODIS summary report given different settings are in the table below where \* indicates an obligate allele.

Max alleles	4	4	2	4	2	
Threshold	0.99	0.999	0.99	0.99	0.99	
Force single genotype ?	Enabled	Enabled	Enabled	Disabled	Disabled	
D3S1358	15,16,17	15,16,17	-	15,16,17	-	
vWA	15,16,18	-	-	15,16,18		
D8S1179	11,12,13,14	11,12,13,14	-	11,12,13,14		
D21S11	29,31,31.2	29,31,31.2	-	29,31,31.2		
D2S441	10,11,13	10,11,13	11	10,11,13	11	
TH01	7,9,9.3	7,9,9.3	-	7,9,9.3	-	
D22S1045	11,15,17	11,15,17	-	11,15,17	-	
D5S818	10,11,13	10,11,13	-	10,11,13	-	
D10S1248	12,14,15,16	12,14,15,16	-	12,14,15,16	-	
D2S1338	17,18,19,22	-	-	17,18,19,22	-	

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 128 of 141

All the alleles within the xml file and CODIS summary report were as expected based on the logic. These tests satisfy the acceptance criteria.

#### 4.23.1. CODIS match threshold

Acceptance criterion: Match statistic precision is expected to 10 sig fig. CODIS inverse match statistic precision is expected to 5 sig fig.

The CODIS match threshold logic follows the formulae:

 $p_i(2-p_i)$ , for a single allele present,

 $\left(\sum p_i\right)^2$ , for more than one allele, with no obligates,

 $p_i\left(p_i + 2\sum_{j,i < j} p_j\right)$ , for more than one allele, one of which is obligate (  $p_i$ )

Where p is the posterior mean frequency of the allele.

A two person mixed GlobalFiler<sup>™</sup> profile was interpreted in STRmix<sup>™</sup>. The inverse match statistic was replicated for contributor one and compared to the CODIS report inverse match statistic. The settings used in the CODIS report were: Max alleles = 4, Genotype Weight Threshold = 99, Match Statistic Threshold = 0, Force Single Genotype = Y. The FBI extended Caucasian allele frequencies were used. The alleles present in the report, the corresponding posterior mean frequencies, locus match statistic and overall profile inverse match statistic are in the table below.

	Allele	S			Posterior Mean Frequencies				
Locus	а	b	С	d	f`а	fb	fc	fd	Match statistic
D3S1358	15	16	17		0.2472	0.2324	0.2102		0.4758444787
vWA	15	18			0.1139	0.2176			0.1098345066
D16S539	12	13			0.3410	0.1632			0.2542704835
CSF1PO	10	11	12	13	0.2521	0.2990	0.3262	0.0718	0.9006883399
TPOX	7	11			0.0004	0.2547			0.06509847754
D8S1179	10	15			0.1089	0.1064			0.04633848146
D21S11	28	30	32.2	33.2	0.1681	0.2323	0.1088	0.0298	0.2905547845
D18S51	16	20			0.1039	0.0249			0.01657599482



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							Date	e of Issue: 29 September 2020	
D2S441	11	14			0.3089	0.2620		0.3258346780	
D19S433	11	12	16.2		0.0002	0.0965	0.0199	0.01359054276	
TH01	6	7			0.2250	0.1731		0.1585219479	
FGA	19				0.0545			0.1060504018	
D22S104 5	11	17	19		0.1435	0.0966	0.0003	0.05775813308	
D5S818	11				0.4077			0.6491997218	
D13S317	11	12			0.3114	0.3090		0.3848593964	
D7S820	8	12			0.1632	0.1410		0.09253613778	
SE33	14.2	21.2		4	0.0025	0.0124		0.0002234511232	
D10S124 8	15	16			0.1731	0.1311		0.09253613778	
D1S1656	11	14			0.0693	0.0915		0.02586405189	
D12S391	17	18			0.1038	0.1754		0.07800105750	
D2S1338	21	22	25		0.0200	0.0298	0.0965	0.02140260631	
STRmix™	match	statistic		1.	543451464	4E-23			
Excel mate	ch statis	stic		1.	543451464	4E-23			
STRmix™	inverse	e match	statistic	6.4	6.4789E+22				
Excel inve	rse mat	ch statis	stic	6.4	4789E+22				

Date of Issue: 29 September 2020

These numbers satisfies the acceptance criterion.

### 4.24. Precision

To demonstrate the precision in the STRmix<sup>TM</sup> v2.8 likelihood ratio, a number of different samples were deconvoluted in STRmix<sup>TM</sup> v2.8 twenty times each and an *LR* was generated for each deconvolution. Due to MCMC variability, results are expected to be reproducible to within one order of magnitude difference.

The samples including the following:

- a fully resolved single source sample (Exercise 1A)
- a partial single source sample (Exercise 1B)
- a 2 person mixture in equal proportions with one assumed contributor (Exercise 1C)
- a 2 person mixture in equal proportions without assumed contributors (Exercise 1C)
- a major:minor 2 person mixture (Exercise 1D)
- a low level single source sample with replicate inputs (Exercise 1E)
- a 2 person mixture run as a variable 2-3 person mixture (varNOC 2-3p)
- a single source profile with replicates from two different DNA profiling kits (Multikit replicates)

The results are plotted below in Figure 4.5 to Figure 4.12. *LR* results are reproducible to within one order of magnitude and hence these samples satisfy the acceptance criteria.



Figure 4.5 Plot of the log(LR) for a fully resolved single source sample (Exercise 1A). As expected there is no variability in the LR as this profile is full resolved.

STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

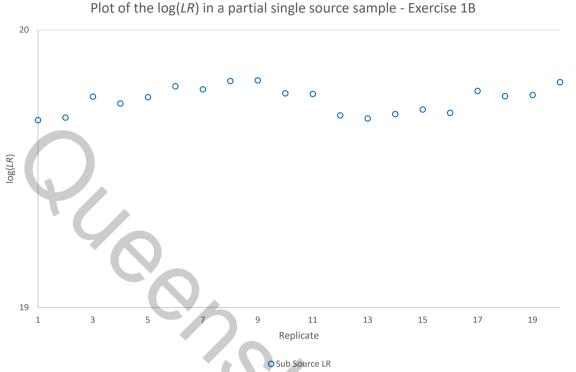


Figure 4.6 Plot of log(LR) of a partial single source sample (Exercise 1B). As expected there is some variability within the LR as this is a partial profile, however the variability is within one

order of magnitude.

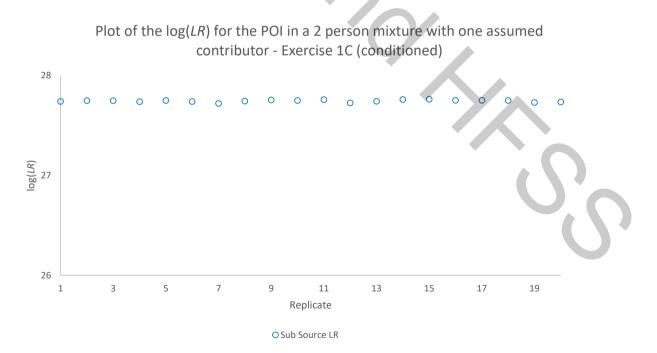


Figure 4.7 Plot of log(LR) for the person of interest in an equal proportion mixture with one assumed contributor (Exercise 1C - conditioned). As expected there is very little variability within the LR when conditioning on one of the contributors to this two person mixture.

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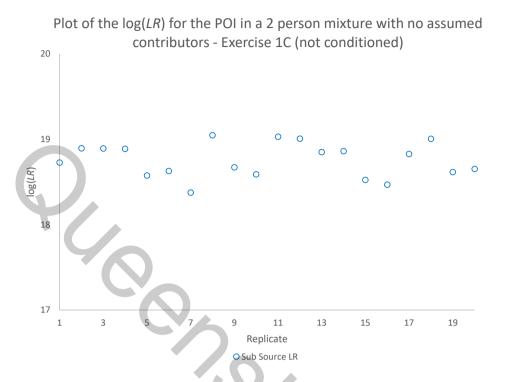


Figure 4.8 Plot of log(LR) for the person of interest in an equal proportion mixture with no assumed contributors (Exercise 1C - not conditioned). As expected there is more variability in the *LR* when there is no assumed contributor in this two person mixture and the mixture proportions are nearly equal. The variability is still within the expected one order of magnitude difference.



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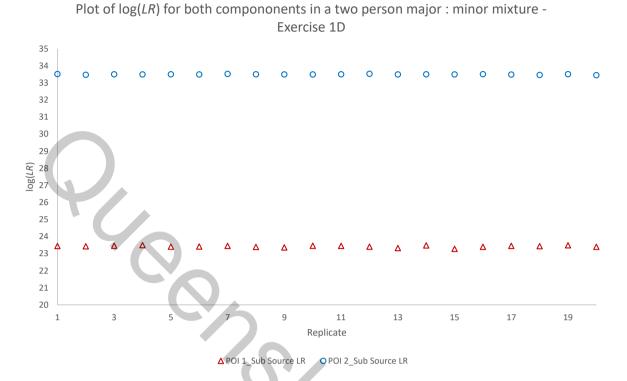


Figure 4.9 Plot of log(LR) for each component in a major:minor 2 person mixture (Exercise 1D). As this mixture can be resolved into clear major and minor components, there is very little variability in the LR and it is well within one order of magnitude difference.

STRMIX ©2020 Institute of Environmental Science and Research Limited Page 134 of 141

STRmix<sup>™</sup> V2.8 Test Report Issued by Institute of Environmental Science and Research Limited Date of Issue: 29 September 2020

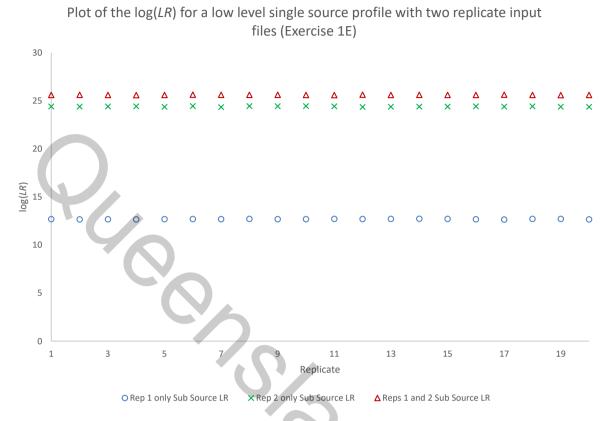
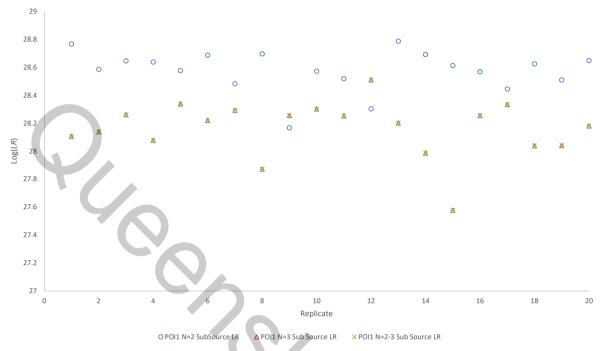


Figure 4.10 Plot of log(LR) for the first and second replicates individually and combined for a low level single source DNA profile - Exercise 1E. There is very little variability in the LR and the results are well within one order of magnitude difference.





Plot of log(*LR*) for POI1 in a 2 person mixture when run as a 2 person and as a 3 person mixture and as a 2 to 3 person mixture - varNOC 2-3p

Figure 4.11 Plot of log(LR) for one person of interest in a 2 person mixture where the sample was run in STRmix<sup>TM</sup> v2.8 with a variable number of contributors (varNOC 2-3p). Variability within each of the three plotted *LR*s is within one order of magnitude difference.

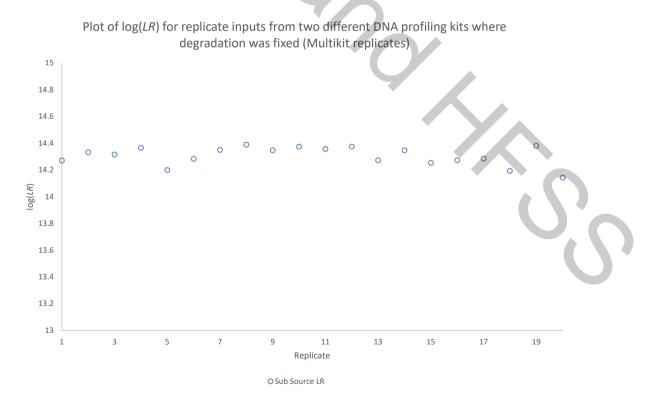


Figure 4.12 Plot of the log(LR) for a single source DNA profile generated in two different DNA profiling kits - Multikit replicates. As expected, the LR is well within one order of magnitude of difference.



# 4.25. Sensitivity and Specificity

A demonstration of the sensitivity and specificity for a range of mixtures interpreted in STRmix<sup>TM</sup> V2.8 was undertaken as per Taylor [2]. With respect to interpretation methods, sensitivity is defined as the ability of the software to reliably resolve the DNA profile of true contributors within a mixed DNA profile for a range of starting DNA templates. The log(*LR*) for contributors ( $H_p$  true) should be high and should trend to 0 as less information is present within the profile. Information includes the amount of DNA from the contributor of interest, conditioning profiles (for example the complainant's profile on intimate samples), replicates and decreasing numbers of contributors (i.e. reduced profile complexity).

Specificity is defined as the ability of the software to reliably exclude non-contributors ( $H_d$  true) within a mixed DNA profile for a range of starting DNA templates. The log(*LR*) should be low and trend upwards to 0 as less information is present within the profile.

Specificity and sensitivity were tested by assigning the *LR* for a number of one-, two-, three-, four-, and five-person profiles for both true contributors and non-contributors. The plots in [2] have been reproduced for these interpreted profiles. The contributors include homozygote and heterozygote alleles and there is varying amounts of allele sharing across the different loci. Given the template amounts allele and/or locus dropout was expected to occur within the profiles containing the lower DNA amounts. A summary of the 323 profiles interpreted is given in the table below.

Each profile was interpreted in STRmix<sup>™</sup> and the results were compared to a database containing the true contributors and non-contributors using the Database Search function within STRmix<sup>™</sup>. The non-contributors were artificially generated.

Using the FBI Extended Caucasian allele frequencies and an  $F_{ST}$  of 0, a sub-source point estimate *LR* was assigned where the propositions considered were:

 $H_p$ : The DNA originated from the database individual and *N*-1 unknown unrelated individuals  $H_d$ : The DNA originated from *N* unknown unrelated individuals Where *N* is the experimental number of contributors to the profile.

	Database size (n <sub>db</sub> )	Number of samples for each experimental design <i>N</i>					Number of samples ( <i>n</i> )	Number of <i>LR</i> s		
Dataset	Dat	1	2	3	4	5	∠ ö	H <sub>p</sub>	H <sub>d</sub>	All
ProvedIt_GF_A	250	0	6	6	6	6	24	84	5916	6000
ProvedIt_GF_B	1050	0	10	10	10	10	40	140	41860	42000
ProvedIt_GF_Conditioned	250	0	7	11	12	17	47	180	11570	11750
ProvedIt_GF_SS	250	145	0	0	0	0	145	145	36105	36250
QIAGEN_Investigator	5	0	35	0	0	0	35	70	105	175
Fusion	230	0	11	11	10	0	32	95	7265	7360
Total		145	69	38	38	33	323	714	102821	103535

#### Table 4.13: Counts of the profiles interpreted.

Plots of the log(*LR*) versus the average peak height (*APH*) per contributor for the singlesource profiles and the two-, three-, four-, and five contributor mixtures are given in the figures below. *APH* was calculated using unmasked, unshared and non-stutter affected alleles for each contributor in the crime scene profile. Where the contributor had completely dropped out of the mixture, an *APH* of half the analytical threshold was applied. The per contributor amount of DNA for  $H_d$  true contributors is taken as the lowest APH of the known contributors.

Exclusions (LR = 0) are plotted as log(LR) = -40. The results of all comparisons are provided below for each *N*.

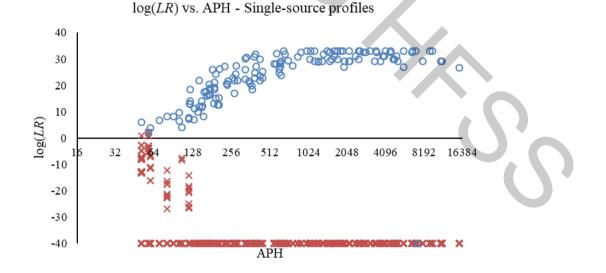


Figure 4.13: Scatter plot of the log(LR) vs. APH for single-source profiles. *LR*s for true donors are plotted as blue circles, *LR*s for non-contributors are plotted as red crosses.

# STRMIX ©2020 Institute of Environmental Science and Research Limited Page 138 of 141

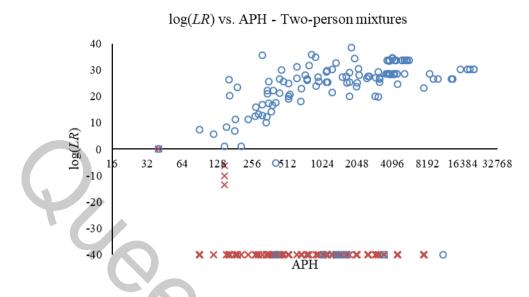


Figure 4.14: Scatter plot of the log(*LR*) vs. APH for two-person profiles. *LR*s for true donors are plotted as blue circles, *LR*s for non-contributors are plotted as red crosses.

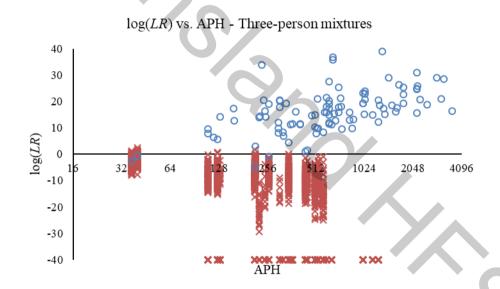
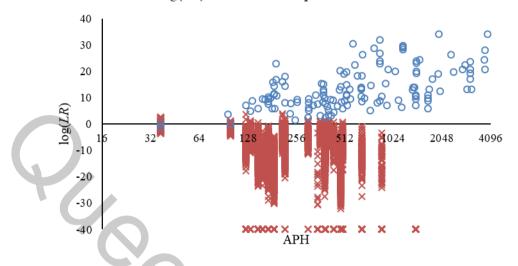


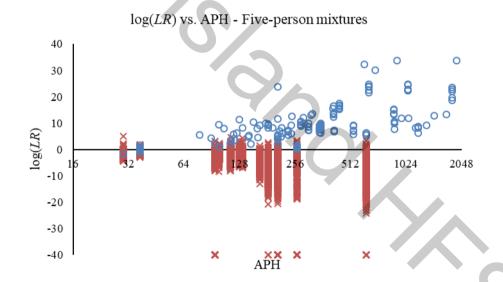
Figure 4.15: Scatter plot of the log(*LR*) vs. APH for three-person profiles. *LR*s for true donors are plotted as blue circles, *LR*s for non-contributors are plotted as red crosses.





log(LR) vs. APH - Four-person mixtures

Figure 4.16: Scatter plot of the log(*LR*) vs. APH for four-person profiles. *LR*s for true donors are plotted as blue circles, *LR*s for non-contributors are plotted as red crosses.



# Figure 4.17: Scatter plot of the log(*LR*) vs. APH for five-person profiles. *LR*s for true donors are plotted as blue circles, *LR*s for non-contributors are plotted as red crosses.

Inspection of the plots shows that as template increases the log(*LR*) distributions for  $H_p$  true and  $H_d$  true are well separated for the single-source profiles and the two-, three-, four-, and five-person mixtures. As the amount of template decreases,  $H_p$  true and  $H_d$  true converge on log(*LR*) = 0. This is intuitive because as profile information is reduced (i.e. template) we would expect our statistic to reliably inform us of such (i.e. tend towards an *LR* of 1 = uninformative). We also see that as the number of contributors to the profile increases (in the plots with the scaled X axes), the log(*LR*)s for  $H_p$  true and  $H_d$  true tend towards 0. This is also intuitive because as profiles become more complex we (generally) expect there to be more genotype combinations possible, leading to a spread of the genotype weights and resulting in lower *LR*s compared to simpler profiles with less ambiguity.

At high template STRmix<sup>TM</sup> correctly and reliably gave a high *LR* for true contributors and a low or exclusionary *LR* for false contributors. At low template and high contributor number STRmix<sup>TM</sup> correctly and reliably reported that the analysis of the sample tends towards uninformative or inconclusive.

There were nine exclusions (LR=0) assigned to known contributors. Six of these exclusions are expected, as they have been identified and diagnosed in other validations. These were due peaks in the profile approaching the saturation threshold which resulted in elevated posterior mean stutter variances.

One of these exclusions was due to a saturated peak that was not labelled in the input file. When this peak was added back into the input file and reinterpreted the LR increased to  $8.089 \times 10^{30}$ .

The two remaining exclusions were due to a single-locus exclusion from the interpretation of the same profile and same contributor. These exclusions were a result of the reference alleles dropping out of the profile. This meant that the dropout of two alleles is needed to be accepted during the deconvolution to produce an inclusionary *LR*.

6

### 5. REFERENCES

- [1] T.R. Moretti, L.I. Moreno, J.B. Smerick, M.L. Pignone, R. Hizon, J.S. Buckleton, J.-A. Bright, A.J. Onorato, Population data on the expanded CODIS core STR loci for eleven populations of significance for forensic DNA analyses in the United States, Forensic Science International: Genetics 25 (2016) 175-181.
- [2] D. Taylor, Using continuous DNA interpretation methods to revisit likelihood ratio behaviour, Forensic Science International: Genetics 11 (2014) 144-153.

